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Voluntary Sovereign Debt Exchanges[*]

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Abstract

We show that some recent sovereign debt restructurings were characterized by (i) the absence of missed debt payments prior to the restructurings, (ii) reductions in the government's debt burden, and (iii) increases in the market value of debt claims for holders of the restructured debt. Since both the government and its creditors are likely to benefit from such restructurings, we label these episodes as "voluntary" debt exchanges. We present a model in which voluntary debt exchanges can occur in equilibrium when the debt level takes values above the one that maximizes the market value of debt claims. In contrast to previous studies on debt overhang, in our model opportunities for voluntary exchanges arise because a debt reduction implies a decline of sovereign default risk. This is observed in the absence of any effect of debt reductions on future output levels. Although voluntary exchanges are Pareto improving at the time of the restructuring, we show that eliminating the possibility of conducting voluntary exchanges may improve welfare from an ex-ante perspective. Thus, our results highlight a cost of initiatives that facilitate debt restructurings.

JEL classification: F34, F41.

Keywords: Sovereign Default, Debt Restructuring, Voluntary Debt Exchanges, Long-term Debt, Endogenous Borrowing Constraints.

1 Introduction

In sovereign debt restructurings the government swaps previously issued debt for new debt. This paper studies restructurings that are characterized by (i) the absence of missed debt payments prior to the restructurings, (ii) reductions in the government's debt burden, and (iii) increases in the market value of debt claims for holders of the restructured debt. Since both the government and its creditors are likely to benefit from such restructurings, we label these episodes as "voluntary" debt exchanges.[1]

We first show evidence that some recent sovereign debt restructurings fit within our definition of a voluntary debt exchange. We then show that a sovereign default model à la Eaton and Gersovitz (1981) can account for voluntary exchanges. Finally, we show that within our framework, and in spite of benefiting all parties involved at the time of the restructuring, voluntary debt exchanges are not always optimal for the government from an ex-ante perspective.

Formally, we analyze a small open economy that receives a stochastic endowment stream of a single tradable good. The government is benevolent, issues long-term debt in international markets, and cannot commit to repay its debt. Sovereign debt is held by risk-neutral foreign investors. We extend this baseline model of sovereign default by allowing for voluntary debt exchanges. We assume that the cost of debt exchanges is stochastic and may be either low (zero) or high. The high cost is assumed to be high enough to prevent the government from launching a voluntary exchange. This assumption intends to capture difficulties in the implementation of voluntary exchanges, including political conflicts and collective action/coordination problems. At the beginning of each period, the debt exchange cost is realized together with the endowment shock. When the cost is low, the government chooses whether to conduct a voluntary debt exchange. If the government conducts a voluntary exchange, the post-exchange debt level is endogenously determined as the outcome of a Nash bargaining game. If the government does

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[1]Our definition of voluntary exchange should not be interpreted as indicating that the participation of all creditors in the exchange was strictly voluntary. First, it is difficult to determine the extent to which governments coerced bondholders into participating in a debt exchange (Enderlein et al., 2012; Zettelmeyer et al., 2012). Furthermore, even when a debt restructuring is beneficial for creditors as a group, individual creditors could benefit from free-riding on the participation of other creditors. For a discussion of collective action problems associated with debt restructuring see Wright (2011) and the references therein.

not conduct a voluntary exchange, it decides whether to declare an outright default. An outright default is followed by a stochastic period of exclusion from capital markets and lower income levels while this exclusion lasts.

Why does a debt relief lead to capital gains for the holders of restructured debt? In our model this occurs because lower debt levels reduce future default risk and thus increase the market value of the bond holders' debt portfolio. Figure 1 shows the market value of debt as a function of the debt stock. If the state of the economy is represented by a point like A, a marginal decline of the debt level is more than compensated by an increase in bond prices, producing an increase in the market value of bond holders' debt portfolio. In this case, both the government and its creditors could benefit from a debt restructuring that reduces the debt stock (for example, to point B).

We show that our model can account for the frequency of voluntary debt exchanges suggested by the findings in Cruces and Trebesch (2013), while still accounting for other features of the data highlighted in the sovereign debt literature. Using a calibration for an archetypical emerging economy, opportunities for voluntary debt exchanges arise in 11 percent of the simulation periods. That is, in 11 percent of the simulation periods the initial debt level is higher than the debt level that maximizes the market value of debt claims. We show that those periods arise after sufficiently negative aggregate income shocks. However, an outright default does not need to be imminent in all those periods—i.e., the government would not necessarily default if the cost of conducting a voluntary exchange was high. The presence of voluntary exchanges in periods without an imminent default is only possible in our model because we allow the government to issue long-term debt.

Among voluntary debt exchanges that prevented an outright default in the period of the exchange, the average capital gain from holdings of the restructured debt is 141 percent. The sovereign enjoys an average debt reduction of 23 percent and an average welfare gain at the times of the exchange equivalent to a permanent consumption increase of 0.6 percent. Among voluntary debt exchanges conducted in periods in which the government would have chosen to stay current on its debt, the average capital gain is 6 percent, the average debt reduction is 7 percent, and the sovereign enjoys an average welfare gain equivalent to a permanent consumption increase of 0.2 percent.

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In spite of the Pareto gains that result at the moment of a voluntary exchange, eliminating the possibility of conducting a voluntary exchange may improve welfare from an ex-ante perspective. The anticipation of future voluntary exchanges leads lenders to offer lower prices (i.e., demand higher yields) when they purchase government bonds. This impairs the possibility to bring future resources forward and is costly for an impatient government that is eager to borrow. This finding highlights a cost of initiatives to facilitate sovereign debt restructurings.

1.1 Related literature

The possibility that a sovereign debtor and its creditors can jointly benefit from a debt reduction is discussed by Froot (1989), Krugman (1988a), Krugman (1988b), and Sachs (1989), among others. They present "debt overhang" models with exogenous debt levels and show that debt relief generates Pareto gains when future debt obligations are high enough to lower aggregate investment. By increasing aggregate investment, a debt relief increases the resources available for creditors to confiscate in the case of a default.[2] In contrast, we present a model in which a debt relief has no effect on future available resources but it weakens incentives to default, producing capital gains for debt holders. Our findings also complement their results by showing that (i) opportunities for Pareto improving debt reliefs arise endogenously in a calibrated model with empirically plausible implications, and (ii) governments may want to prevent debt reliefs from an ex-ante perspective, even when these reliefs generate Pareto gains at the time of the debt restructuring.

Aguiar et al. (2009) study a model with endogenous debt levels in which a government that cannot commit to future actions may accumulate excessive debt claims causing a debt overhang effect on investment. The optimal allocation under no commitment lies on the constrained efficient frontier, which rules out the possibility of mutually beneficial debt reductions once the sovereign is in a situation of debt overhang. In contrast, we focus on debt reliefs that produce

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[2]It should be noted that creditors face difficulties when trying to confiscate assets of a defaulting country (see, for instance, Panizza et al., 2009 and Hatchondo and Martinez, 2011). In many countries (including the U.S.), there are legal procedures that creditors may follow once individuals or corporations renege on their debt. Creditors rely on the enforcement of domestic courts to get—partially—repaid. However, it is difficult for courts to enforce government payments.

Pareto Gains at the time of the restructuring even though they may not be optimal from an ex-ante perspective.

In the recent quantitative papers on sovereign default that followed Aguiar and Gopinath (2006) and Arellano (2008), the possibility of engaging in mutually beneficial debt relief is ruled out by assumption. We extend the baseline model by allowing for this possibility which enables us to account for some of the heterogeneity among debt restructurings. We show that the model can account for voluntary debt exchanges while still accounting for other features of the data emphasized in the sovereign default literature.

The rest of the article proceeds as follows. Section 2 argues that some recent sovereign debt restructurings fit within our definition of a voluntary debt exchange. Section 3 introduces the model. Section 4 discusses the calibration. Section 5 presents the results. Section 6 concludes.

2   Voluntary debt exchanges in the data

This Section presents evidence that shows that some recent sovereign debt restructurings fit within our definition of a voluntary debt exchange. Recall that we define a voluntary debt exchange as a debt restructuring such that (i) the government does not miss any debt payment prior to the restructuring, (ii) there is a decline in the government's debt burden, and (iii) bond holders enjoy capital gains from participating in the restructuring.

A significant fraction of sovereign debt restructurings were not preceded by missed debt payments. For instance, Cruces and Trebesch (2013) study 180 sovereign debt restructurings between 1970 and 2010 and show that in 77 of these 180 restructurings the government did not miss debt payments prior to the restructuring.

Most sovereign debt restructurings are characterized by debt reductions. Measuring the implied debt reductions requires assumptions that allow for the comparison of debt instruments with different characteristics. One measure of debt reduction is the "haircut". There are different haircut measures. Recently, the one proposed by Sturzenegger and Zettelmeyer (2005) has been widely accepted and used (see for example Cruces and Trebesch, 2013):

$$Haircut = 1 - \frac{\text{Present value of debt obtained in the restructuring}}{\text{Present value of debt surrendered in the restructuring}}, \qquad (1)$$

where both present values are computed using the yields of the debt received at the moment of the restructuring. Using this measure, Cruces and Trebesch (2013) find that only 6 of the 180 restructurings they study present negative haircuts. This indicates that governments benefited from a debt reduction in the remaining 174 cases.[3]

We find that some recent sovereign debt restructurings that occurred in the absence of missed debt payment were characterized by a lower debt burden for the government and a higher market value of the debt claims of holders of restructured debt. We compute capital gains for holders of restructured debt using bond prices to determined the value of the debt portfolio of these bond holders, before and after the restructurings:

$$\text{Capital gain} = \frac{\sum_{i=J+1}^{J+I} q_i(T) \times B_i}{(1+r)^{T-t} \sum_{j=1}^{J} q_j(T-t) \times B_j} - 1, \qquad (2)$$

where T denotes the period for which prices of bonds obtained in the exchange are available, $T - t$ denotes the period for which we compute the market value of the portfolio surrendered in the exchange, $q_i(t)$ denotes the unit price of bonds of type i in period t, and B_i denotes the number of outstanding bonds of type i. Equation (2) refers to a case in which the debt portfolio $(B_1, ..., B_J)$ is exchanged for the debt portfolio $(B_{J+1}, ..., B_{I+J})$.

We are interested on the effect of debt reductions on the market value of the creditors' debt portfolio. Therefore, we compute the market value of the portfolio surrendered in the exchange at different pre-restructuring dates in an attempt to control for the effect of the anticipation of the restructuring on this market value. If the success of the exchange is perfectly anticipated, at the time of the exchange, the value of the portfolio surrender by lenders should be equal to the value of the portfolio they receive in the exchange.

Table 1 presents creditors' capital gains from holding restructured debt in six recent episodes.[4]

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[3]It should be noted that debt reductions are only a partial measure of the benefits accrued to sovereign debtors. Debt restructurings typically benefit debtor governments by smoothing and/or extending the maturity profile of debt payments.

[4]Secondary market bond prices make it easier to compute the market values of debt for bond debt restructurings than for bank debt restructurings. Cruces and Trebesch (2013) show that out of the 180 debt restructurings that

The Table reports capital gains up to six months before the exchange announcement. Details on the construction of debt portfolio values are presented in the Appendix.

Figure 2 presents the market value of debt portfolios for the six exchanges in Table 1. Figure 2 and Table 1 show that four of the debt exchanges under consideration produced capital gains when we considered the price of surrendered bonds at the moment of the informal announcement. These are the episodes in Dominican Republic, Pakistan, Ukraine, and Uruguay. Figure 2 shows a common pattern in the value of the portfolio surrendered in the exchange for these four episodes: this value increases in the weeks preceding the exchange.

All episodes in Table 1 are characterized by debt reductions, as measured by positive haircuts (the Table presents the haircuts computed by Cruces and Trebesch, 2013 and Zettelmeyer et al., 2012). The restructurings in Belize and Greece had the highest haircuts among the six episodes we consider. This is consistent with the capital losses we measure for those episodes.

Notice that the capital gain measured as in equation (2) equals the opposite of the haircut measure (1) for the case in which present values are computed using the yield implicit in each bond price—and thus the present value of payments promised in a bond is given by its price. Thus, the key difference between the haircut and capital gain measures presented in Table 1 is in the yield used to compute the present value of debt surrendered in the restructuring: for the haircut measure we use the yields implicit in the price of bonds obtained in the restructuring, and for the capital gain measure we use the yields implicit in the price of bonds surrendered in the restructuring. This indicates that for the four restructurings that present capital gains—and therefore fit within our definition of voluntary exchanges—the implied yields in sovereign bond prices decreased after these restructurings. This could be due to a decline in country risk implied by each restructuring, which is consistent with our theory of voluntary debt exchanges.

took place between 1970 and 2010, 162 were bank debt restructurings and only 18 were bond debt restructurings. Out of the 18 bond debt restructurings, 9 occurred without the government missing debt payments prior to the restructuring. These are the recent restructurings in Belize, Dominica, Dominican Republic, Grenada, Kenya, Moldova, Pakistan, Ukraine, and Uruguay. Table 1 presents data for the five of these restructurings for which we found bond price data, plus the 2012 Greece restructuring.

	Belize	Dom. Rep.	Greece	Pakistan	Ukraine	Uruguay
First date with all bond prices	2/21/07	5/18/05	3/9/12	n.a.	3/30/00	9/23/03
Extended deadline for participating	2/15/07	7/20/05	4/4/12	12/13/99	3/15/00	5/22/03
Original deadline for participating	1/26/07	5/11/05	3/8/12	12/13/99	3/15/00	5/15/03
Launch of the exchange	12/18/06	4/20/05	2/24/12	11/15/99	2/14/00	4/10/03
Formal exchange announcement	8/2/06	3/30/05	2/24/12	8/15/99	2/4/00	3/11/03
Informal exchange announcement (IA)	7/15/06	4/16/04	7/21/11	1/15/99	1/14/00	2/18/03
Capital gains with price of surrendered portfolio at						
First date with all bond prices	0.10	0.08	0.02	n.a	-0.01	0.01
Extended deadline for participating	0.11	0.11	-0.27	0.05	0.14	0.10
Original deadline for participating	0.10	0.08	0.03	0.05	0.14	0.12
Launch of the exchange	0.05	0.04	-0.01	0.04	0.18	0.30
Formal exchange announcement	-0.05	0.06	-0.01	0.00	0.24	0.28
Informal exchange announcement	-0.11	0.24	-0.59	0.07	0.48	0.22
One month before IA	-0.14	0.27	-0.60	0.22	0.49	0.04
Three months before IA	-0.19	0.10	-0.66	-0.02	0.39	0.34
Six months before IA	0.05	n.a.	-0.69	-0.38	-0.12	0.38
Acceptance rate	0.98	0.94	0.97	0.99	0.99	0.95
Haircut	0.24	0.05	64.8	0.15	0.18	0.10

Table 1: Capital gains from holdings of restructured debt in recent sovereign debt restructurings.

3 Model

There is a single tradable good. The economy receives a stochastic endowment stream of this good y_t, where

$$\log(y_t) = (1 - \rho)\,\mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$.

The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right),$$

where $\mathbb{E}$ denotes the expectation operator, β denotes the subjective discount factor, and the utility function is assumed to display a constant coefficient of relative risk aversion denoted by γ. That is,

$$u\left(c\right) = \frac{c^{(1-\gamma)} - 1}{1 - \gamma}.$$

As in Hatchondo and Martinez (2009), we assume that a bond issued in period t consists of a perpetuity with geometrically declining coupon obligations. In particular, a bond issued in period t promises to pay one unit of the good in period $t+1$ and $(1-\delta)^{s-1}$ units in period $t+s$, with $s \geq 2$.

The government cannot commit to repay its debt. We refer to events in which the government reneges on its debt as outright defaults. As in previous studies, the cost of an outright default does not depend on the size of the default. Thus, when the government defaults, it does so on all current and future debt obligations.[5] Following previous studies, we also assume that the recovery rate for outright defaults is zero.

An outright default triggers exclusion from credit markets for a stochastic number of periods. Income is given by $y - \phi\left(y\right)$ in every period in which the government is excluded from credit

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[5]This is consistent with the behavior of defaulting governments in reality. Sovereign debt contracts often contain acceleration and cross-default clauses. These clauses imply that after a default event, future debt obligations become current (see IMF, 2002a).

markets. As in Arellano (2008), we assume that it is proportionally more costly to default in good times. This is a property of the endogenous default cost derived by Mendoza and Yue (2012). As in Chatterjee and Eyigungor (2012), we assume a quadratic loss function for income during a default episode $\phi(y) = max\{d_0 y + d_1 y^2, 0\}$. They show that this function allows the equilibrium default model to match the behavior of the spread in the data.

Lenders are risk neutral and discount future payoffs at the rate r. Bonds are priced in a competitive market inhabited by a large number of identical lenders, which implies that bond prices are pinned down by a zero expected profit condition.

Our departure from the baseline setup is to allow for voluntary debt exchanges. We present a stylized model of voluntary exchanges. We intend to capture the difficulties in implementing a voluntary restructuring by assuming that the cost of debt exchanges is i.i.d. and may take a low (zero) or a high value. The high cost is assumed to be sufficiently elevated to make it optimal for the government to not launch a voluntary exchange when the cost is high.[6]

When the cost of a voluntary exchange is zero, the government chooses whether to conduct an exchange. If the government conducts a voluntary exchange, it reduces its debt level. We assume that the post-exchange debt level is endogenously determined in a Nash bargaining game in which the government and bond holders negotiate over the debt level.[7]

The government cannot commit to future outright default, exchange, and borrowing decisions. Thus, one may interpret this environment as a game in which the government making the exchange, default, and borrowing decisions in period t is a player who takes as given the exchange, default and borrowing strategies of other players (governments) who will decide after t. We focus on Markov Perfect Equilibrium. That is, we assume that in each period, the government's equilibrium exchange, default and borrowing strategies depend only on payoff-relevant state variables. Krusell and Smith (2003) discuss the possibility of multiple Markov perfect equi-

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[6]Voluntary exchanges may be prevented because of collective action problems (Wright, 2011), which are sometimes mitigated by the inclusion of collective action clauses, minimum participation thresholds, and exit consents in debt contracts (see IMF, 2013). We interpret a low exchange cost shock as representing circumstances in which collective action problems can be overcome. We calibrate the probability of such shock targeting the frequency of voluntary exchanges. Since voluntary exchanges in our model are only conducted when there is no cost for the sovereign, our results may provide an upper bound of the benefits derived from debt exchanges.

[7]We rule out the possibility that bond holders can extract side payments from the goverment in order to accept a restructuring proposal.

libria in infinite-horizon economies. In order to avoid this problem, we solve for the equilibrium of the finite-horizon version of our economy. That is, we increase the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium policy functions as an approximation of the stationary equilibrium policy functions.

3.1 Recursive formulation

Let b denote the number of outstanding coupon claims at the beginning of the current period. Let $\theta \in \{L, H\}$ denote the exchange-cost shock, where L (H) denotes the low (high) value of the shock. Let V denote the value function of a government that is not currently in default. This function satisfies the following functional equations:

$$V(b, y, L) = \max \left\{ V^E(b, y), V^P(b, y), V^D(y) \right\}, \tag{3}$$

and

$$V(b, y, H) = \max \left\{ V^P(b, y), V^D(y) \right\}, \tag{4}$$

where V^E denotes the continuation value when the government launches a debt exchange, V^P denotes the continuation value when the government pays its debt obligations (and does not launch an exchange), and V^D denotes the continuation value when the government declares an outright default.

The value function of paying current debt obligations and not launching an exchange is represented by the following functional equation:

$$V^P(b, y) = \max_{b' \geq 0, c} \left\{ u\left(c\right) + \beta \mathbb{E}_{(y', \theta')|y} V(b', y', \theta') \right\}, \tag{5}$$

subject to

$$c = y - b + q(b', y) \left[b' - (1 - \delta)b \right],$$

where $b' - (1 - \delta)b$ represents current debt issuances, and q denotes the price of a bond at the end of a period. Let $\hat{b}^P$ denote the government's borrowing rule when it stays current on its debt obligations.

The value function when the government declares an outright default satisfies the following functional equation:

$$V^D(y) = u(y - \phi(y)) + \beta \mathbb{E}_{(y', \theta')|y} \left[(1 - \psi) V^D(y') + \psi V(0, y', \theta') \right], \tag{6}$$

where ψ denotes the probability of regaining access to capital markets. Let $\hat{d}$ denote the government's default strategy. The function $\hat{d}$ takes a value of 1 when the government declares an outright default and takes a value of 0 when it stays current on its debt.

Finally, the value function when the government launches a debt exchange satisfies:

$$V^E(b, y) = V^P(\hat{b}^E(b, y), y), \tag{7}$$

where $\hat{b}^E(b, y)$ denotes the post-exchange debt level. That is, in a restructuring period the government services the debt agreed on the restructuring and then it issues (or buys back) debt.

The post-exchange debt level is endogenously determined in a Nash bargaining game in which bond holders' bargaining power is constant over time and denoted by α, namely:

$$\hat{b}^E(b, y) = \underset{b^E}{\operatorname{argmax}} \left\{ \left[MV(b^E, y) - MV(b, y) \right]^\alpha \left[V^P(b^E, y) - Max \left\{ V^P(b, y), V^D(b, y) \right\} \right]^{1-\alpha} \right\}$$
$$s.t. \quad MV(b^E, y) - MV(b, y) \geq 0, \text{ and}$$
$$V^P(b^E, y) - Max \left\{ V^P(b, y), V^D(b, y) \right\} \geq 0,$$

where $MV(b, y)$ denotes the market value of debt claims outstanding at the beginning of the period. If it is optimal for the government to default when it starts with a state vector (b, y), the market value is zero. If it is optimal to repay, the market value equals the sum of current coupon payments and the value at which bond holders can sell their bond portfolio. Formally,

$$MV(b, y) = b \left[1 - \hat{d}(b, y) \right] \left[1 + (1 - \delta) q \left(\hat{b}^P(b, y), y \right) \right]. \tag{8}$$

The surplus that each party obtains in the restructuring consists of the difference between the continuation value if the restructuring takes place and the continuation value without restructuring. If a restructuring does not take place, the market value of bond holders' debt claims

is $MV(b, y)$ and the continuation value for the government is $Max\left\{V^P(b, y), V^D(b, y)\right\}$. If a restructuring that brings the debt level to b^E takes place, the market value of bond holders' debt claims is $MV(b^E, y)$ and the continuation value for the government is $V^P(b^E, y)$.

Let $\hat{e}$ denote government's exchange strategy. The function $\hat{e}$ takes a value of 1 when the government chooses to conduct a debt exchange. Recall we assume that the high cost of an exchange is such that $\hat{e}(b, y, H) = 0$ for all (b, y).

The assumption that bond holders price bonds in competitive markets implies that

$$
\begin{aligned}
q(b', y)(1 + r) \;=\; & \mathbb{E}_{(y', \theta')|y}\left\{\hat{e}\left(b', y', \theta'\right)\frac{\hat{b}^E(b, y)}{b'}\left[1 + (1 - \delta)\, q(\hat{b}^P(\hat{b}^E(b', y'), y'), y')\right]\right. \\
& \left. + \left[1 - \hat{e}\left(b', y', \theta'\right)\right]\left[1 - \hat{d}\left(b', y', \theta'\right)\right]\left[1 + (1 - \delta)\, q(\hat{b}^P(b', y'), y')\right]\right\}, \quad (9)
\end{aligned}
$$

where $\frac{\hat{b}^E(b, y)}{b'} \leq 1$ denotes the number of bonds received in the exchange per bond surrendered. Notice that the model equivalent of the definition of haircut presented in the introduction is given by $1 - \frac{\hat{b}^E(b, y)}{b'}$.

3.2 Equilibrium definition

A *Markov Perfect Equilibrium* is characterized by

1. rules for voluntary exchanges $\hat{e}$, default $\hat{d}$, and borrowing $\hat{b}^P$,

2. and a bond price function q,

such that:

i. given a bond price function q, the policy functions $\hat{d}$, $\hat{e}$, and $\hat{b}^P$ solve the Bellman equations (3), (4), (5), (6), and (7).

ii. given policy rules $\hat{d}$, $\hat{e}$, and $\hat{b}^P$, the bond price function q satisfies condition (9).

4 Calibration

Table 2 presents the benchmark values given to the parameters in the model. A period in the model refers to a quarter. The coefficient of relative risk aversion is set equal to 2, the risk-free

interest rate is set equal to 1 percent, and the discount factor is set equal to 0.975. These are standard values in quantitative business cycle and sovereign default studies. The average duration of sovereign default events is three years ($\psi^d = 0.083$), in line with the duration estimated by Dias and Richmond (2007).

Risk aversion	γ	2
Risk-free rate	r	0.01
Discount factor	β	0.975
Probability of reentry after default	ψ	0.083
Probability of high exchange shock	π	0.02
Income autocorrelation coefficient	ρ	0.94
Standard deviation of innovations	σ_ϵ	1.5%
Mean log income	μ	$(-1/2)\sigma_\epsilon^2$
Debt duration	δ	0.033
Income cost of defaulting	d_0	-1.01683
Income cost of defaulting	d_1	1.18961
Creditors bargaining power	α	1

Table 2: Benchmark parameter values.

We use Mexico as reference for the calibration. Mexico is an archetypical emerging economy that is commonly used as reference in previous work that studies these economies (see, for example, Aguiar and Gopinath, 2007, Neumeyer and Perri, 2005, and Uribe and Yue, 2006). The parameter values that govern the income process are estimated using GDP data from the first quarter of 1980 to the fourth quarter of 2011.

We set $\delta = 3.3$ percent. With this value, bonds have an average duration of 5 years in the simulations, which is roughly the average debt duration observed in Mexico according to Cruces et al. (2002).[8]

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[8]We use the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = \frac{1+r^*}{\delta+r^*}$,

We allow creditors to have full bargaining power during the debt exchange bargaining game. In terms of Figure 1, this means that if the government conducts a debt exchange in a period characterized by point A, it lowers its debt level to the one depicted by point B. It is often argued that friendly restructurings maximize creditors' recovery rate subject to bringing the debt to a "sustainable" level. This seems consistent with our baseline of $\alpha = 1$. In Subsection 5.6 we present comparative statics with respect to α.

We calibrate the values of d_0, d_1, and π (the probability of a high exchange cost) targeting the average levels of spread and debt between 1995 (do to data availability) and 2011, and a share of voluntary debt exchanges to default episodes of 30 percent. Debt restructurings that fit within our definition of voluntary exchange are likely to be considered sovereign defaults. For instance, this is often the case for the four restructurings described in Section 2 (for example, Cruces and Trebesch, 2013 describe these episodes as defaults). Credit-rating agencies consider a "technical" default an episode in which the sovereign makes a restructuring offer that contains terms less favorable than the original debt, leaving considerable ambiguity on how to determine whether terms offered to investors were less favorable. Usually, the haircut definition presented in Section 2 is used as a measure of investor losses.[9] This interpretation of the haircut would be consistent with considering all voluntary exchanges as sovereign defaults, as we do for interpreting simulation results. Cruces and Trebesch (2013) report that 43 percent of the default episodes they study occur before the government defaults on debt payments. As we show in Section 2, not all these episodes result in capital gains for investors. Thus, 43 percent is an upper bound for the share of voluntary debt exchanges to default episodes.

5 Results

We first show that the model simulations reproduce features of the data. We then discuss the opportunities for voluntary debt exchanges in the simulations and the role of long-term debt

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where r^* denotes the constant per-period yield delivered by the bond. Using a sample of 27 emerging economies, Cruces et al. (2002) find an average duration of foreign sovereign debt in emerging economies—in 2000—of 4.77 years, with a standard deviation of 1.52.

[9]In fact, the objective of Sturzenegger and Zettelmeyer (2005) when they introduce this definition is to measure investors' losses.

in generating these opportunities, the gains at the moment of a voluntary exchange, and the ex-ante gains from avoiding voluntary debt exchanges. Finally, we present comparative statics with respect to creditors' bargaining power in determining the post-exchange debt level.

The recursive problem is solved using value function iteration. The approximated value and bond price functions correspond to the ones in the first period of a finite-horizon economy with a number of periods large enough to make the maximum deviation between the value and bond price functions in the first and second period small enough. We solve for the optimal debt level in each state by searching over a grid of debt levels and then using the best point on that grid as an initial guess in a nonlinear optimization routine. The value functions V^E, V^P, and V^D, and the bond price function q are approximated using linear interpolation over y and cubic spline interpolation over debt levels.[10] We use 50 grid points for both debt and income. Expectations are calculated using 75 quadrature points for the income shock.

In order to compute the sovereign spread implicit in a bond price, we first compute the yield i an investor would earn if it holds the bond to maturity (forever in the case of our bonds) and no default is ever declared. This yield satisfies

$$q_t = \frac{1}{(1+i)} + \sum_{j=1}^{\infty} \frac{(1-\delta)^j}{(1+i)^{j+1}}.$$

The sovereign spread is the difference between the yield i and the risk-free rate r. We report the annualized spread

$$r_t^s = \left(\frac{1+i}{1+r}\right)^4 - 1.$$

Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate, i.e., $b_t(1+r)(\delta+r)^{-1}$, where $\delta = 1$ for one-period bonds. We report debt levels as a percentage of annualized income.

We generate 500 samples of 500 periods each. We extract the last 120 periods of samples in which the last default was observed at least 20 periods prior to the beginning of those 120 periods. With the exception of the default rates, moments in the simulations correspond to the averages

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[10]Hatchondo et al. (2010) discuss the advantages of using interpolation and solving for the equilibrium of a finite-horizon economy.

over those samples. The frequency of outright defaults and debt restructurings are computed using all simulation data. Table 3 presents the calibration targets and the corresponding moments in the simulations. The Table shows that the simulations approximate the targets reasonably well.

	Targets	Simulations
Mean Debt-to-GDI	44	42
Mean r_s	3.40	3.35
Voluntary debt exchanges / defaults	0.30	0.27

Table 3: Calibration targets and simulation moments. Moments for the simulations correspond to the mean value of each moment in 500 simulation samples.

5.1 Non-targeted simulation moments

Table 4 reports non-targeted simulation moments. The table shows that allowing for voluntary exchanges does not compromise the default model's ability to account for key features of business cycles statistics in emerging economies (Aguiar and Gopinath, 2006, Aguiar and Gopinath, 2007, Neumeyer and Perri, 2005, and Uribe and Yue, 2006 documents those features). Our model is able to account for a volatile and countercyclical spread that leads to more borrowing in good times than in bad times, as reflected by a volatile and countercyclical trade balance.

5.2 Voluntary debt exchanges

Opportunities for voluntary debt exchanges arise on average in 11 out of 100 simulation periods. That is, in 11 percent of the simulation periods the initial debt level b is higher than the debt level that maximizes the market value of debt (denoted by $\bar{b}(y)$). There are two reasons why this may happen. First, b may be higher than $\bar{b}(y)$ because of a sufficiently negative income shock. Even when the government chooses $\hat{b}^P(b,y) < \bar{b}(y)$, it may still be that next period $\hat{b}^P(b,y) > \bar{b}(y')$. The left panel of Figure 3 depicts the market value of debt claims for two

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	Mexico	Simulations
$\sigma\left(r_s\right)$	1.6	1.4
$corr\left(r_s, y\right)$	-0.5	-0.7
$\sigma(c)/\sigma(y)$	1.3	1.3
$\rho(tb, y)$	-0.7	-0.8
$\rho(r_s, tb)$	0.8	0.9

Table 4: Non-targeted simulation moments. The standard deviation of x is denoted by $\sigma\left(x\right)$. The coefficient of correlation between x and z is denoted by $corr\left(x, z\right)$. Moments for the simulations correspond to the mean value of each moment in simulation samples.

income levels. The Figure shows that declines in income shift the market value curve towards the origin and reduce the debt level that maximizes MV. This occurs because, as is standard in models of sovereign default that generate countercyclical spreads, lower income levels imply higher default probabilities. This is illustrated in the right panel of Figure 3.

Furthermore, even when the government starts the period with $b < \bar{b}(y)$, it may choose to end a period with a debt level higher than the one that maximizes the debt market value. Figure 4 presents an example of these situations in the simulations. We find however that such examples are only observed in 3 percent of the simulation periods. Furthermore, as illustrated in Figure 4, b' is typically very close to b in these periods.

Figure 5 presents the equilibrium outright default and voluntary exchange policies. Figure 5 shows that the government prefers to participate in a debt exchange only when aggregate income takes a sufficiently low value (grey and black areas). The income threshold at which the government is indifferent between participating and not participating in a debt exchange is higher than the income threshold at which it is indifferent between an outright default and repaying current debt obligations. This also means that all outright defaults in the simulations would have been avoided with a low cost of voluntary exchange. In addition, Figure 5 shows that for some state realizations the government would choose to conduct a voluntary exchange even if the impossibility of an exchange would not have triggered an outright default (grey area). This is consistent with the presence of opportunities for capital gains at debt levels that would

not trigger an outright default, as illustrated in the left panel of Figure 3.

5.3 The role of long-term debt

Table 5 presents the benchmark simulations (when the government issues only long-term debt) and simulations of the model with only one-period debt ($\delta = 1$). When the government only issues one-period debt, the spread and default rates are significantly lower than in the baseline calibration (see Hatchondo and Martinez, 2009). In addition, the share of default episodes that can be described as voluntary debt exchanges drop from 27 percent in our benchmark to 2 percent with one-period debt. This is the case because in the one-period-bond version of the model, all voluntary exchanges prevent an outright default. Notice that with one-period bonds, the market value of the debt stock at the beginning of a period is given by $MV(b,y) = \left[1 - \hat{d}(b,y)\right] b$. Therefore, a debt reduction can only increase this market value (making a voluntary exchange possible) when the debt reduction prevents an outright default. As illustrated in Figures 3 and 5, this is not the case with long-term debt: at the beginning of a period, the debt level may be higher than the one that maximizes the debt market value (generating the possibility of a voluntary exchange) even if the initial debt level would not lead the government to default in the current period. In fact, only 5 percent of voluntary exchanges in our benchmark simulations prevent an outright default in the period of the exchange.

	Benchmark	One-period bonds
Mean Debt-to-GDP	42.4	32.3
Mean r_s	3.35	0.14
Outright defaults per 100 years	2.21	0.14
Voluntary debt exchanges per 100 years	0.81	0.003

Table 5: Simulations in benchmark model and with one-period debt.

Furthermore, the government would never choose a debt level higher than the one that maximizes the debt market value in the one-period-bond version of the model. Such a choice is

only optimal when the issuance proceeds are different from the market value (as in our baseline calibration with long-term debt). This is not the case with one-period bonds.[11] Choosing a debt level higher than the one that maximizes the debt market value implies choosing a debt level such that the effect of additional borrowing on the market value is negative. If the government repays its debt and does not launch a voluntary exchange, the first order condition for debt accumulation is given by:

$$\frac{d\left(q\left(b',y\right)b'\right)}{db'} = -\frac{\beta}{u'(c)}\frac{d\left(\mathbb{E}_{(y',\theta')|y}V\left(b',y',\theta'\right)\right)}{db'} + (1-\delta)\,b\frac{d\left(q\left(b',y'\right)\right)}{db'},\tag{10}$$

assuming the value and bond price functions are differentiable.

The left hand side of equation (10) represents the marginal effect of the last bond issued in the period on the market value of debt claims outstanding at the end of the period. In the model simulations, the first term of the right hand side is always positive (the government is worse off when it starts a period with a higher debt level) while the second term is negative (bond prices are lower when the government chooses a higher debt level). Thus, the government only chooses debt levels higher than the one that maximizes the market value when that second term is low enough. This is never the case in the one-period-bond version of the model ($\delta = 1$), where the second term is equal to zero.

5.4 Gains at the moment of a voluntary debt exchange

Table 6 reports the average capital gain, haircut, and welfare gain from voluntary exchanges. The Table shows that, consistently with our definition, theses exchanges produce gains for lenders (as indicated by positive capital gains) and the government (as indicated by positive haircuts and welfare gains). We measure welfare gains as the constant proportional change in consumption that would leave domestic consumers indifferent between not having the option to conduct a voluntary exchange ($\theta = H$) and having this option ($\theta = L$):

$$\left(\frac{V(b,y,L)}{V(b,y,H)}\right)^{\frac{1}{1-\gamma}} - 1.$$

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[11]When the government issues debt, it is concerned about the market value of the debt claims that it is issuing (i.e., the issuance proceeds) and not about the overall value of the debt stock (see Hatchondo et al., 2011). With one-period debt, debt issued in the current period constitutes the total debt stock.

	Exchanges that prevent an outright default	Other exchanges
Average capital gain	141%	6%
Average haircut	23%	7%
Average welfare gain	0.6%	0.2%

Table 6: Gains from a voluntary debt exchange. Capital gains for exchanges that prevent an outright default are computed using the market value of the debt stock the quarter before the exchange.

5.5 Ex-ante gains from avoiding voluntary debt exchanges

Figure 6 represents the proportional consumption compensation that leaves domestic residents indifferent between living in an economy without debt exchanges ($\pi = 0$) and moving to an economy with debt exchanges ($\pi = 0.02$). A positive number means that domestic residents better off without debt exchanges. The welfare gain is evaluated at a debt level equal to the mean debt observed in the simulation and before the government learns the current exchange cost. Formally, the welfare gain is computed as:

$$\left(\frac{\pi_0 V(b, y, L; \pi_0) + (1 - \pi_0)V(b, y, H; \pi_0)}{\pi_1 V(b, y, L; \pi_1) + (1 - \pi_1)V(b, y, H; \pi_1)} \right)^{\frac{1}{1-\gamma}} - 1,$$

for $\pi_0 = 0$, and $\pi_1 = 0.02$.

Figure 6 shows that eliminating the possibility of conducting a voluntary exchange may improve welfare for sufficiently high income realizations. This is so even though voluntary exchanges are Pareto improving at the time of the restructuring (see Table 6). At those income levels, it is not optimal for the government to conduct a voluntary exchange in the current period even if the exchange cost is low. Besides, it is unlikely that the government will conduct a voluntary exchange in the near future. On the contrary, the possibility of conducting voluntary exchanges improves welfare at sufficiently low income levels because the government will conduct one if the exchange cost is low. The average welfare gain from eliminating the possibility of conducting voluntary exchanges is 0.015 percent. The ex-ante welfare loss from allowing for debt exchanges is consistent with governments' reluctance to issue easier to restructure debt. Of course, since our benchmark probability of a low cost exchange cost is only 2 percent, welfare gains from lowering

	Benchmark	No exchanges
Mean Debt-to-GDP	42.5	42.3
Mean r_s	3.4	3.1
$\sigma(r_s)$	1.4	1.2
Outright defaults per 100 years	2.2	2.1
Voluntary debt exchanges per 100 years	0.8	0

Table 7: Simulations when the government cannot conduct voluntary exchanges.

this probability to zero are modest.

The possibility of conducting debt exchanges may reduce welfare because it increases the borrowing cost. Creditors are forward looking and forecast that when voluntary exchanges are possible, the set of states in which debt claims are going to be partially written off is larger. The anticipation of that has an adverse effect on the current-period borrowing set available to the government, as illustrated in the left panel of Figure 7, and impairs the government's ability to bring future resources forward. At the same time, the possibility of debt exchanges has good insurance properties because it enables the government to write off debt claims in states with sufficiently low income realizations. This is illustrated in the right panel of Figure 7.

Table 7 reports the main statistics in the two economies. In equilibrium, the lower bond prices faced by the government in the economy with debt exchanges induces the government to issue more debt to bring resources forward. This leads to a slightly higher spread and frequency of outright defaults. Given that outright defaults are costly events, a higher frequency of outright defaults has a negative effect on welfare. Thus, the overall negative welfare effect of debt exchanges suggests that the benefit from a smoother consumption profile is not large enough to compensate for the reduction of intertemporal trading opportunities and the higher frequency of outright defaults.

	$\alpha = 1$	$\alpha = 0.7$	$\alpha = 0.5$
Mean Debt-to-GDP	42.5	42.5	42.5
Mean r_s	3.4	3.5	3.6
Outright defaults per 100 years	2.2	2.3	2.3
Voluntary debt exchanges per 100 years	0.8	0.9	0.9
Average haircut (in %)	8	11	13
Welfare gain from eliminating VDE (% of cons.)	0.015	0.027	0.033

Table 8: Simulations in economies with different creditors' bargaining power.

5.6   Comparative statics with respect to creditors' bargaining power

Table 8 shows that, as one would expect, reducing creditors bargaining power (up to 0.5) increases the average haircut in voluntary exchanges, increases the frequency of voluntary exchanges and, consequently, increases the welfare gain from eliminating the possibility of conducting these exchanges. The Table also shows that changes in bargaining power do not affect significantly other implications of the model, which may not be surprising since voluntary exchanges are infrequent.

Figure 8 illustrates the haircut after a voluntary debt exchange as a function of income for two economies: the benchmark ($\alpha = 1$) and an economy with $\alpha = 0.7$. The Figure shows that, when creditors do not have full bargaining power, the post-exchange debt level presents a discontinuity. This discontinuity arises because, as illustrated in Figure 3, the market value presents a discontinuity, dropping to zero in states at which the government would chose an outright default if the cost of conducting a voluntary exchange is high. A lower market value of debt enables the government to increase the debt relief that it can extract from creditors.[12]

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[12]The discontinuity in Figure 8 also implies a discontinuity in the value function V^E. This creates problems when interpolation methods are used to compute expectations. We solve this problem approximating two auxiliary functions for the post-exchange debt level (and thus two auxiliary functions for the value of conducting an exchange): one function assumes that the government does not default in the current period in any state (and follows the optimal strategies in the future), and the other function assumes that the government always defaults in the current period if there is no voluntary exchange (and follows the optimal strategies in the future). We then use the first (second) function to interpolate for states in which the government repays (defaults).

6 Conclusions

This paper first documents that some recent sovereign debt restructurings were not preceded by missed debt payments and resulted in both sovereign debt reductions and capital gains for the holders of restructured debt. The paper then proposes a model that accounts for such restructurings. In contrast with standard debt overhang arguments, our model delivers Pareto gains from debt restructurings even when debt reductions have no effect on future output levels. These gains arise only because of the decline of sovereign risk (i.e., the odds of a future outright default) implied by the debt reduction. In our model, voluntary exchanges are possible after negative income shocks, but do not require an outright default to be imminent. Most voluntary exchanges in the model simulations happen in periods in which the government would not choose to declare an outright default. The paper also shows that in spite of the Pareto improvement at the time of voluntary exchanges, the government may prefer an environment in which conducting these exchanges is more difficult.

Overall, the paper presents an attempt to enrich the understanding of gains from renegotiation between creditors and debtors but also highlights a time inconsistency problem in the government's decision of undertaking renegotiations.[13] The discussion of these issues would certainly benefit from a more thorough analysis of the possibility of Pareto gains in past sovereign debt restructuring experiences, and richer theories that, for instance, endogenize renegotiations both after outright defaults and in voluntary debt exchanges.[14]

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[13]Hatchondo et al. (2012) and Hatchondo et al. (2011) highlight a time inconsistency problem in the government's borrowing decision. Hatchondo and Martinez (2013) highlight a time inconsistency problem in the government's debt maturity choice.

[14]Arguably, mechanisms that facilitate voluntary debt exchanges may also facilitate the restructurings that follow after an outright default. We abstract from that possibility. Benjamin and Wright (2008) and Yue (2010) model debt renegotiation but only after an outright default.

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A   Appendix

A.1   Calculation of capital gains for debt restructurings

We next describe the calculation of capital gains presented in Figure 2 and Table 1. For each restructuring, we report the average capital gain for all bonds for which we found the necessary price data. Capital gains were calculated for each restructured bond. The weight of each restructured bond capital gain in the average capital gain is given by the outstanding nominal debt restructured for each bond at the moment of the exchange, divided by the total outstanding nominal debt restructured. When comparing bond prices from different dates, bond prices are discounted using the underlying discount rate at the moment of each restructuring, as reported by Cruces and Trebesch (2013) and Zettelmeyer et al. (2012). We present next a brief description of the six exchanges studied in this paper (see also Díaz-Cassou et al., 2008; Das et al., 2012; IMF, 2002b; Rambarram and Bhagoo, 2005; and Robinson, 2010).

A.2   Belize

It has been argued that the 2007 restructuring in Belize became apparent in July 2006 with the appointment of financial advisors with extensive expertise in restructurings. On February 2007 Belize completed the restructuring of its private external debt. Various instruments with a face value of US$571 million (half of GDP), were exchanged for a single bond. The new instrument was a floating exchange bond with maturity in 2029 that extended the debt profile 14 years at no nominal cost for investors. Collective action clauses were used to facilitate the restructuring of a bond governed under New York law. A creditor committee and a communication program were created for the restructuring. We compute capital gains for debt representing 36 percent of the outstanding face value.

A.3   Dominican Republic

Negotiations with the Paris Club in April 2004 allowed the Dominican government to reschedule its bilateral debt service due in 2004, but required the government to seek comparability of treatment with its private creditors. On August 16, 2004, at the Inauguration of the president,

President Fernández announced an emergency plan for the first 100 days of his administration, emphasizing the need of honouring the commitment to the terms of the Paris Club Agreement. After some delay, the Dominican Senate approved on March 30, 2005, the authorization for the executive branch to proceed with the exchange offer. From early on, consultations took place with investors. We compute capital gains for debt representing 55 percent of the outstanding face value.

A.4 Greece

On July 11, 2011, the "Statement by The Heads of State or Government of the Euro Area and EU Institutions" declared that Greece required as an exceptional and unique solution "Private Sector Involvement" on a debt exchange. On October 26, the idea was reinforced by the Euro Summit Statement. On February 21, 2012, both Greece government and a committee of twelve major creditors announced the main terms and conditions of a plan to restructure over 200 billion Euros in privately held Greek bonds. Formal invitations were sent on February 24. The Greek debt restructuring process was as a hybrid between a negotiation led by a steering committee of creditors and a take-it-or-leave-it debt exchange offer (see Zettelmeyer et al., 2012). The English law eligible bonds had a Collective Action Clause (CAC). The Greek law eligible bonds (91 percent of the total eligible titles) did not originally have it but a retrofit CAC was implemented after the consent of a qualified majority. For 96 percent of eligible bonds, the package of new instruments comprised: (i) One and two year notes issued by the European Financial Stability Facility, amounting to 15 per cent of the old debt's face value; (ii) 20 new government bonds maturing between 2023 and 2042, amounting to 31.5 per cent of the old debt's face value and (iii) a GDP-linked security. We compute capital gains for debt representing 96 percent of the outstanding face value.

A.5 Pakistan

Pakistan negotiated a Paris Club restructuring in January of 1999 which required the country to seek comparable debt relief from private creditors, and in particular, to restructure its international bonds. By July, the government had signed a rescheduling with commercial banks. On

November 15, Pakistan launched a bond exchange, ahead of a Paris club deadline that required it to show "progress" in negotiations with bondholders by the end of 1999. The exchange involved swapping three bonds. All three were to be exchanged for a new bond. There was no nominal haircut. In fact, holders of the two bonds with the shorter average life received slightly more in nominal terms than under the original instruments. The new bond was also expected to be more liquid than the old bonds. We compute capital gains for debt representing 75 percent of the outstanding face value.

A.6 Ukraine

In years 1998 and 1999, a succession of selective restructurings with specific creditors was completed in order to bridge mounting liquidity needs. Although restructurings of 1998-99 provided some immediate cash flow relief, they also created large payments obligations for 2000 and 2001. On January 14, 2000, Reuters spread information about the intention of Ukraine to announce the decision to reschedule its foreign debt. On February 4, 2000, Ukraine launched a comprehensive exchange offer involving all outstanding commercial bonds. Creditors could choose between two 7-year coupon amortization bonds denominated either in Euros or U.S. dollars, to be issued under English law. The exchange offer established a minimum participation threshold of 85 percent among the holders of bonds maturing in 2000 and 2001. We compute capital gains for debt representing 29 percent of the outstanding face value.

A.7 Uruguay

On February 18, 2003, President Batlle confirmed the possibility of a debt exchange. Meetings with IMF country authorities towards the debt restructuring took place the previous week. The external debt restructuring was formally announced on March 11, 2003. Creditors were not asked to accept a reduction on the principal. The exchange targeted all traded debt (about half of total sovereign debt) and offered most bondholders a choice between two options. A combination of the use of exit consents to change the non-payment terms of the old bonds and regulatory incentives contributed to the high acceptance rate, 93%, constituted by 98% of local security holders and approximately 85% of international security holders. We compute capital gains for

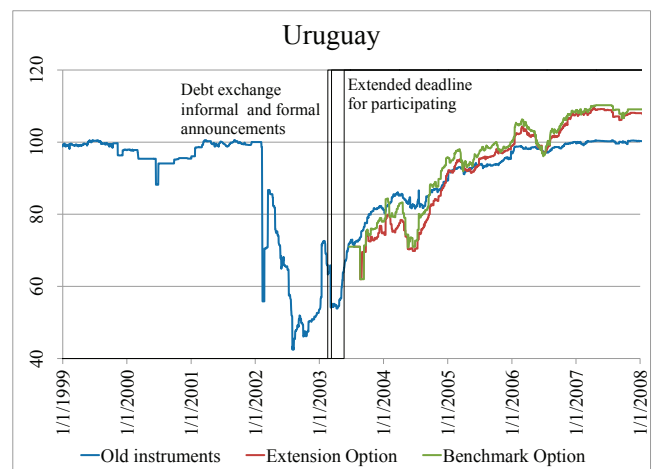debt representing 51 percent of the outstanding face value.

The figure plots the market value of debt against Debt (b), showing the curve $b \times q(b, s_t)$. Point B is at the peak of the curve and point A is slightly to the right of the peak.
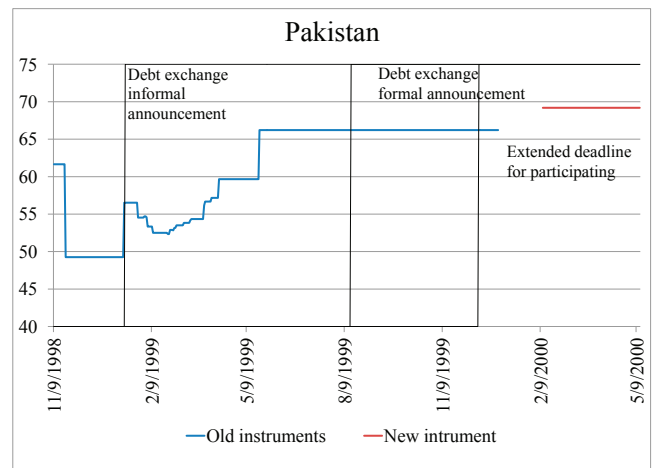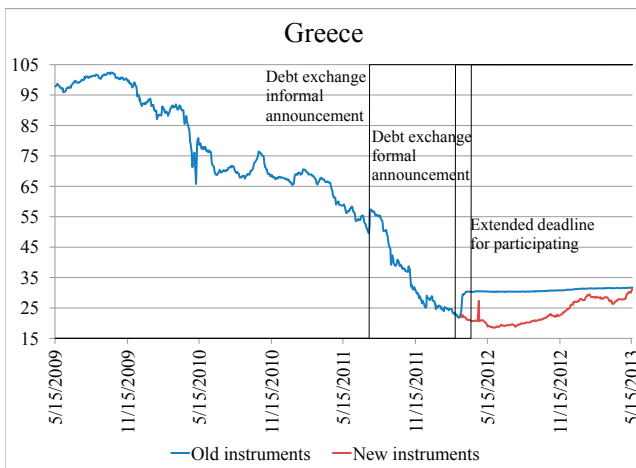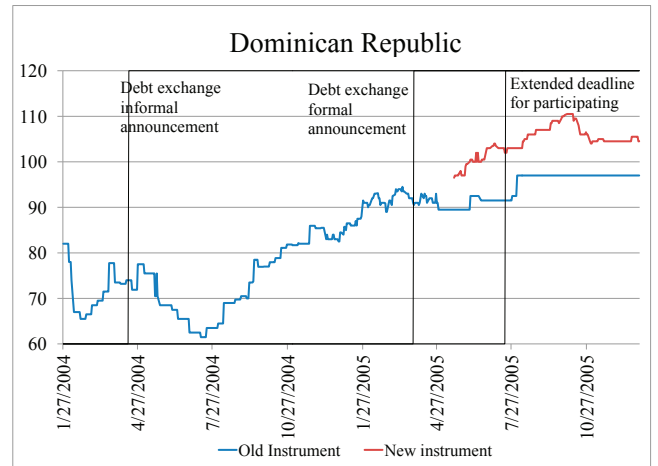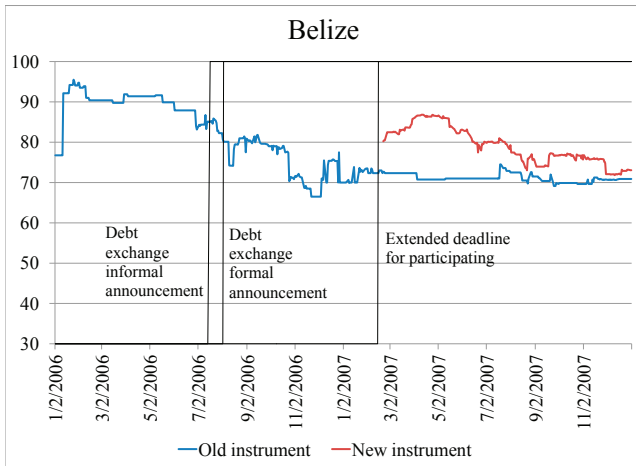
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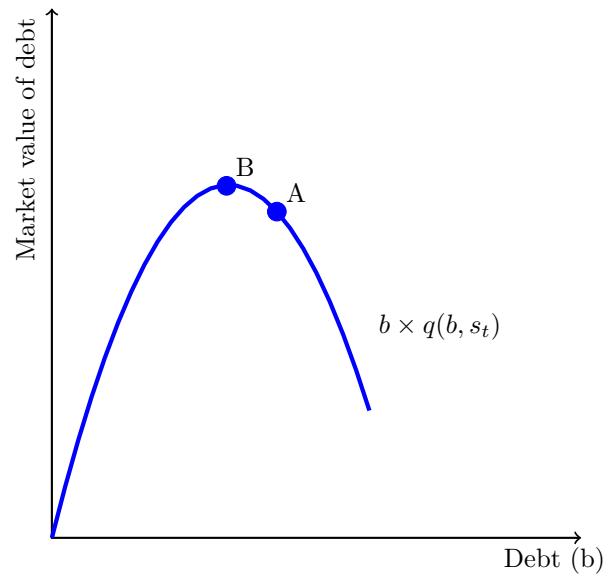
Figure 2: Market value of pre-restructuring and post-restructuring debt portfolios. Bond prices are expressed as a percentage of the par value of pre-restructuring bonds.
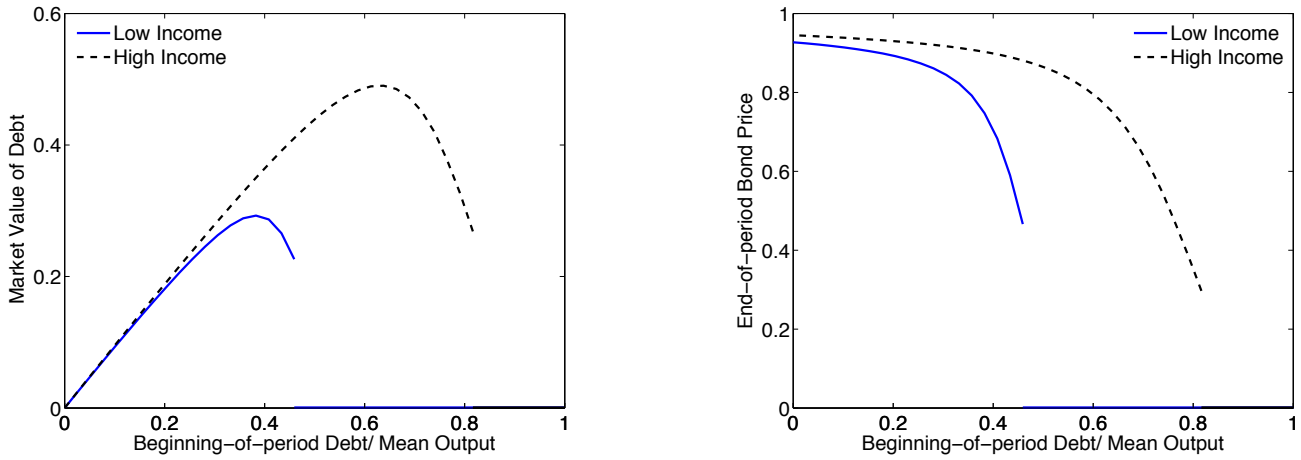
Figure 3: Market value of the debt stock. The left panel plots $MV(b, y)$. The right panel plots the end-of-period bond price when the cost of an exchange is high: $[1 - \hat{d}(b, y, H)]q(\hat{b}^P(b, y), y)$. The Figure refers to income levels one standard deviation below and above the unconditional mean as Low income and High income. .
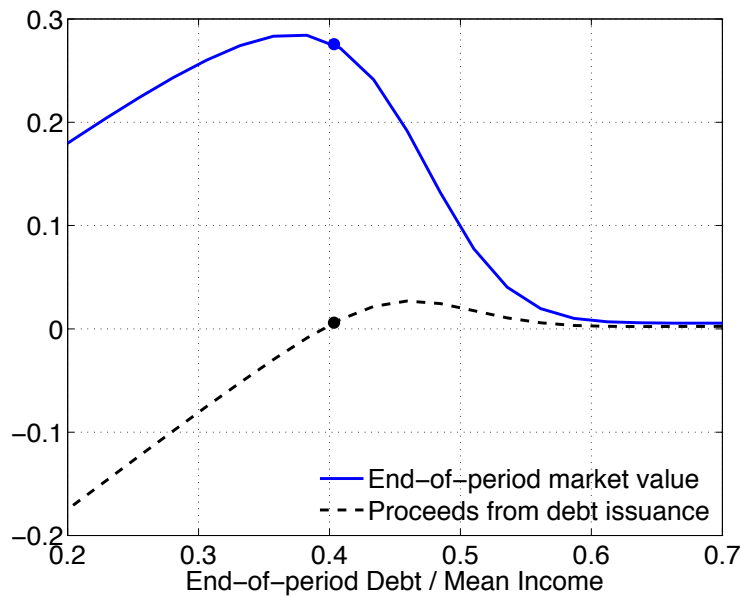
Figure 4: The end-of-period market value of debt claims is defined as $q(b', y)b'$. The proceeds of debt issuances consist of $q(b', y)[b' - (1 - \delta)b]$. The graph assumes that the state is characterized by an initial debt level of 41 percent of mean income and that the current income realization is one standard deviation below the mean. The solid dots represent the optimal decision of a sovereign that did not exchange debt in the period.
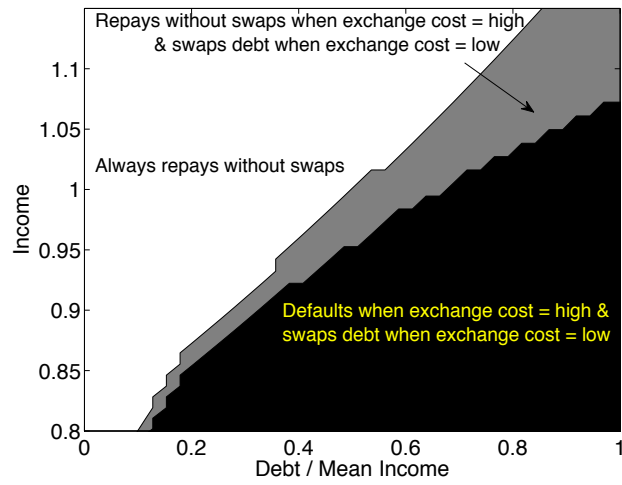
Figure 5: Equilibrium voluntary exchange and outright default policies. For each debt level, the figure presents the maximum income level for which the government would choose to declare an outright default (if the cost of a debt exchange is high) or participate in a debt exchange (if the cost of a debt exchange is low).
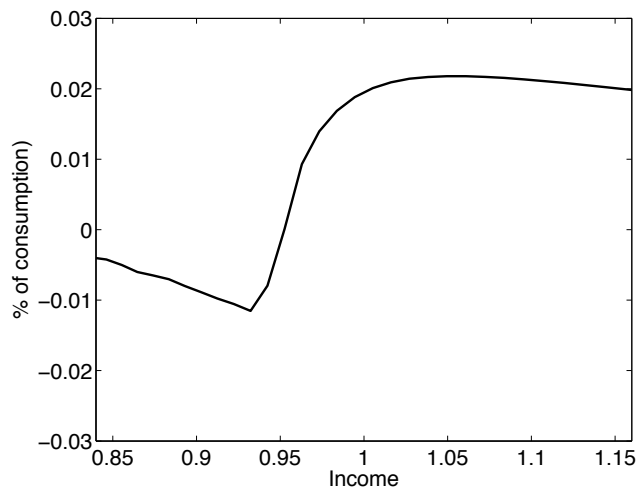

Figure 6: Welfare gains derived from eliminating the possibility of voluntary exchanges. Welfare gains are computed for a debt level equal to the mean debt level in the simulations of the benchmark, and before current exchange cost realization.
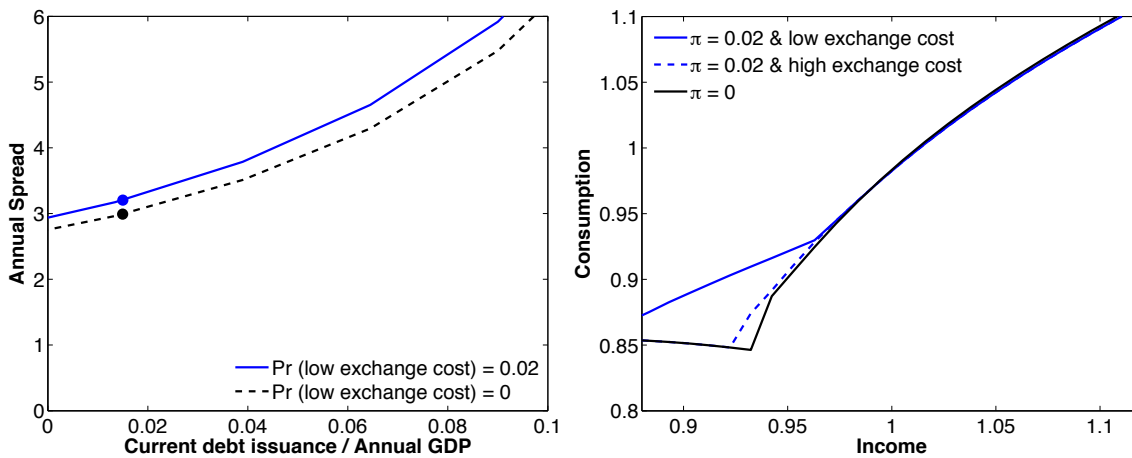
Figure 7: The left panel represents the schedule of borrowing and interest rates available to the government for two economies: the benchmark economy ($\pi = 0.02$) and the economy without swaps ($\pi = 0$). The panel assumes that current income equals the mean income. The solid dots represent optimal choices when the current debt level equals the mean debt in the benchmark simulations. The right panel depicts equilibrium consumption choices when the initial debt level is equal to the mean debt observed in the benchmark simulations.
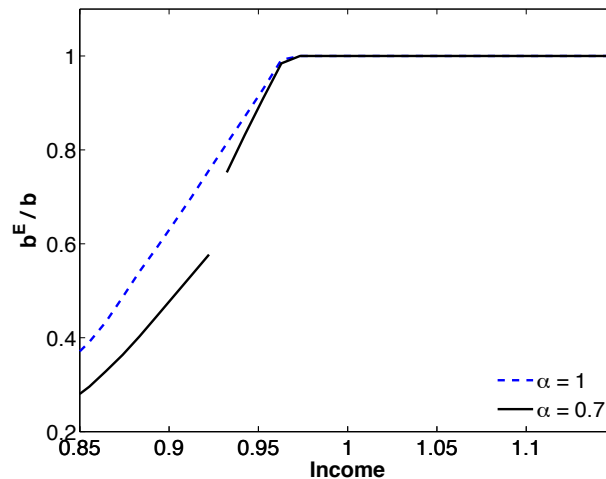

Figure 8: Voluntary debt exchange recovery rate as a function of income and creditors' bargaining power for a debt level equal to the mean debt observed in the simulations of the benchmark economy.