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Trade Policies, Firm Heterogeneity, and Variable Markups

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Abstract

We study unilateral trade liberalization in the model with variable markups. First, we show that the effect of falling per unit trade costs depends on the use of the “outside good” assumption: in its presence trade liberalization reduces welfare at home, and raises it otherwise. Second, we derive the optimal values of import tariffs for the large and small economies and show that in both cases protection is a desirable policy. Finally, we demonstrate that compared to the models with constant markups, variable markups in our setting result in negative pro-competitive effects, reducing gains from trade.

1 Introduction

Recently, there has been a surge in international trade models with imperfect competition and heterogenous firms. In addition to the expansion in product variety studied by Krugman (1980), these models offer new channels, through which trade affects welfare. A number of papers headed by the seminal work of Melitz (2003) highlight the mechanism of self-selection of more efficient firms into exporting. In the presence of such mechanism trade liberalization leads to reallocation of resources towards more efficient firms, improving average productivity in liberalizing countries and potentially raising welfare there.¹ Most of these papers rely on the assumptions of monopolistic competition and constant elasticity of substitution (CES) preferences as in Dixit and Stiglitz

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¹One has to be careful while interpreting productivity growth as a welfare-improving change. See, for example, Demidova and Rodríguez-Clare (2009), who show that an increase in average productivity (as a result of an export subsidy) does not necessarily mean an increase in welfare. In fact, welfare falls in their model.

(1977), which imply constant markups charged by firms. While being extremely convenient from the analytical point of view, constant markups are at odds with the empirical evidence.² Moreover, models with constant markups also ignore so called pro-competitive gains from trade that arise due to the presence of variable markups.³ This important channel has become a focus of recent literature, which allows for variable markups by deviating from the assumption of monopolistic competition and/or incorporating non-CES preferences.⁴ The pro-competitive mechanism of trade in these models is two-fold. First, at the firm level, trade liberalization intensifies foreign competition, reducing market power of local producers and forcing them to decrease their markups.⁵ Second, at the industry level, trade liberalization has ability to affect the markup distribution, reducing its dispersion. As shown, for example, by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), lower markup dispersion, in turn, increases total factor productivity. The reason is that a higher markup dispersion is associated with more extensive distortion that arises due to variability of revenue productivity (the product of physical productivity and a firm's output price) across firms. By reducing this misallocation distortion, trade liberalization can potentially raise welfare.⁶

On the other hand, Edmond, Midrigan and Xu (forthcoming) and Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012) point out to the possibility of negative pro-competitive effects of trade liberalization. The reason is that while trade liberalization leads to labor reallocation towards more productive firms, i.e., exporters, these firms could internalize the drop in trade costs and charge higher markups. As a result, whether trade liberalization leads to welfare gains or losses depends on a joint movement of labor reallocation and markup distribution.

Given the new insights from the recent trade literature on variable markups, what can we say

²See Tybout (2003) for the overview of empirical evidence on variation in markups across firms.

³The importance of these gains is emphasized in Edmond, Midrigan and Xu (forthcoming), who show that the size of these gains is especially large in the presence of significant misallocations and weak cross-country comparative advantage in individual sectors.

⁴The examples of the settings with variable markups include, amongst others, the model of monopolistic competition in Melitz and Ottaviano (2008), the Cournot competition model in Atkeson and Burstein (2008) and Edmond, Midrigan and Xu (forthcoming), the Bertrand competition setting in Bernard, Eaton, Jensen and Kortum (2003), de Blas and Russ (2015) and Holmes, Hsu and Lee (2013).

⁵Edmond, Midrigan and Xu (forthcoming) provide a brief survey of existing empirical literature, which, with a notable exception of De Loecker, Goldberg, Khandelwal and Pavcnik (2012), supports the claim that trade liberalization reduces firms' markups.

⁶See Restuccia and Rogerson (2013) for an excellent survey of the literature on misallocations and their relationship with productivity.

about the desirable trade policies? Is full trade liberalization optimal or is there a need for some sort of protection? Does it matter what form trade liberalization takes, i.e., is there difference between a reduction in import tariffs and a fall in non-tariff trade barriers such as per unit trade costs? The main goal of this paper is to partially fill the gap in the literature and provide tractable analytical results for the optimal trade policy in the two-country model of monopolistic competition with firm heterogeneity and variable markups. As the base model for our analysis, we rely on the work by Melitz and Ottaviano (2008), a well-known extension of Melitz (2003) that incorporates endogenous markups by using the linear demand system with horizontal product differentiation developed by Ottaviano, Tabuchi and Thisse (2002). As pointed out by Nocco, Ottaviano and Salto (2014), who study the closed economy case of this model, endogenous markups create an additional within-sector misallocation: more productive firms do not pass on their entire cost advantage to consumers by absorbing part of it in the markup and end up selling too small quantities compared to the optimal levels. The opposite happens with high cost producers, whose varieties end up being oversupplied. Hence, variable markups in the presence of firm heterogeneity result in misallocation distortion. The natural question is what the effect of unilateral trade liberalization is in the presence of such distortion.

The main prediction of Melitz and Ottaviano (2008) is that a fall in per unit trade costs of foreign exporters reduces welfare of the liberalizing country, while raising welfare of its trading partner. However, the authors rely on the “*outside good*” assumption, i.e., in addition to the Melitz (2003) type sector, they incorporate another one, usually called an “outside good” sector, that produces a homogenous good that is freely traded, made one-to-one from labor in all countries, and used as numeraire. The use of the outside good assumption is very popular in many extensions of the Melitz (2003) model as well as in other works in trade literature.⁷ One well-known advantage of this assumption is significant simplification of analytical derivations, since the outside good pins down wages across all countries exogenously. Its second advantage is the opportunity to study cross-sectoral inefficiencies. Yet, such an assumption comes with a price. First, it excludes an important channel, through which trade affects welfare, namely, an income effect. Second, it adds an extra distortion to the model, since there is no markup in the outside good sector, whereas in the Melitz (2003) type sector producers charge prices above their marginal costs. As pointed out by Bhagwati (1971), the presence of distortions can result in the breakdown of Pareto-optimality of laissez-faire. Hence, it is not surprising that the use of an outside good can potentially distort

⁷See, for example, Yeaple (2005), Ederington and McCalman (2008), Ossa (2011), etc.

the impact of various changes on welfare.

In the first part of this paper, we demonstrate that this is exactly what happens in the case of Melitz and Ottaviano (2008). In particular, we show that once the “outside good” assumption is dropped from their model, the analysis can be carried out in a way similar to Demidova and Rodríguez-Clare (2013), i.e., one can use the help of a simple figure that summarizes the key relationships in the model: the equilibrium results in two conditions that relate wage with the cost cutoff for domestic sellers in the Home economy. Using this figure, it is straightforward to show that in the absence of the outside good, unilateral trade liberalization by Home is welfare-increasing for both the Home and Foreign economies. Moreover, we show that this result also holds in the case of a small Home economy, which we model in line with Demidova and Rodríguez-Clare (2009).

Next, we use the extension of the Melitz and Ottaviano (2008) model without an outside good to study a non-wasteful import tariff, i.e., the one that generates tariff revenues. The reason we want to contrast import tariffs with per unit trade costs is that in the case of the Melitz (2003) model with CES preferences and constant markups, the comparison of the results for trade costs (see Felbermayr and Jung, 2012, and Demidova and Rodríguez-Clare, 2013) with the results for import tariffs (see Demidova and Rodríguez-Clare, 2009, and Felbermayr, Jung and Larch, 2013) shows that the type of trade barriers matters: while full liberalization is optimal in the case of per unit trade costs, if the import tariff is modeled as non-wasteful, protection becomes an optimal policy for the Home government, whether the Home economy is modelled as a small or large one. The intuition behind these results is that in addition to the familiar terms of trade externality, imperfect competition together with firm heterogeneity gives a rise to the markup and consumption-surplus distortions. The former comes from the fact that there are markups charged by local producers, whereas prices of imported goods are equal to opportunity costs, which provides an argument in favor of trade protection, since it reduces the markup distortion by shifting consumption towards domestic goods. The second distortion arises because consumers at Home ignore the effect of their spending on a number of imported Foreign varieties, which turns out to be below its optimal level. The implication of the second distortion is that trade liberalization becomes a desirable policy, since it helps to induce foreign producers’ entry into the local market and increase the number of varieties they supply. The interplay between two distortions results in a positive value of the import tariff for both small and large Home economies, so that the markup distortion prevails.

Hence, the natural question is whether in our model with non-CES preferences the outcome of the analysis of imports tariffs would be different from the one for per unit trade costs. We answer

this question in the second part of the paper, where we derive the optimal values of an import tariffs for two cases of the small and large Home economies and show that, as in the case with CES preferences, protection is an optimal policy. In particular, a strictly positive import tariff maximizes welfare at Home, with the level of protection being higher in the case of a larger economy due to the terms of trade externality. Therefore, although the presence of variable markups creates an additional misallocation distortion, it does not outweigh the effect of other distortions in the model and does not shift the policy choice from protection to full trade liberalization. This result seems to be in line with Bagwell and Lee (2015), who use Melitz and Ottaviano (2008) to study the impact of import tariffs and export subsidies in the case of two symmetric countries. Unlike Melitz and Ottaviano (2008), they also consider revenue-generating import tariffs and, amongst other results, show that a marginal increase in the tariff imposed by the Home country raises welfare there at the cost of its trading partner. However, the main difference between their and our results (other than asymmetric countries and non-marginal changes in the tariff levels in our model) is that Bagwell and Lee (2015) maintain the assumption of the outside good. And although it seems that welfare results in their model resemble ours, the mechanisms behind these results are not the same. For instance, their model generates a *Metzler Paradox*: as Home increases its import tariff, its average price increases, while that abroad falls. This is not the case in our model, since both Home and Foreign average prices rise together, meaning that the popular outside good assumption is not innocuous, and one has to be careful while interpreting the results obtained when it is used.

Overall, our analysis allows to conclude that welfare implications of unilateral trade liberalization are qualitatively the same in the settings with constant and variable markups. What role then do the variable markups play, if any? In the last part of the paper we show that whether unilateral trade liberalization comes in form of falling trade costs or reductions in import tariffs, the presence of variable markups results in the negative pro-competitive effect. In the former case, a small fall in trade costs of foreign exporters causes labor at home to reallocate towards goods that are oversupplied, i.e., those that have a low markup, which, as pointed out by Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012), leads to smaller welfare gains. In the case of import tariffs, we show that if the level of protection is low enough to begin with, its further reduction raises the average markup faced by consumers at home, which, as discussed by Edmond, Midrigan and Xu (forthcoming), implies smaller welfare gains as well. Thus, variable markups in our setting reduce potential gains from trade.⁸ That is why it is not surprising to see that, as we show in the last

⁸It is worth mentioning that the absence of pro-competitive gains in our model might be partially explained by

part of the paper, the degree of intervention in our model is higher compared to that in the Melitz (2003) model with constant markups.

The paper proceeds as follows. Section 2 derives the equilibrium conditions for two cases of the large and small Home economies. Per unit trade costs and import tariffs are introduced into the model and analyzed in Sections 3 and 4, respectively. The role of variable markups is studied in Section 5. Section 6 offers concluding remarks.

2 The Model

In this Section we first modify the Melitz and Ottaviano (2008) model by dropping the outside good assumption, i.e., wages everywhere will be determined endogenously, and derive the equilibrium conditions for the case of two large economies. Then, we show how to modify these conditions for the small Home economy case.

2.1 Two Large Economies

2.1.1 Demand

There are two countries, Home and Foreign, of size L and L^* , respectively. We will denote the Foreign country's parameters with asterisks. Wage in the Foreign country, w^* , is normalized to unity. By dropping the outside good from the Melitz and Ottaviano (2008) model,⁹ we get the following household's maximization problem with a non-separable quadratic utility function (α can be normalized to 1, but we keep it as it is to ease the comparison with Melitz and Ottaviano, 2008):

$$U = \alpha \int_{\Omega} q^c(i) di - \frac{1}{2} \gamma \int_{\Omega} (q^c(i))^2 di - \frac{1}{2} \eta \left(\int_{\Omega} q^c(i) di \right)^2 \quad s.t. \quad \int_{\Omega} p(i) q^c(i) di = w,$$

the assumption about the cost distribution, which is specified as Pareto. As a result of this assumption, a fall in trade costs, for example, leaves the local average markups as well as their dispersion unaffected. Feenstra (2014), who studies the model with non-CES preferences, shows that once the support of the distribution becomes bounded, other channels of pro-competitive gains from trade are back to work, affecting the average markup as well as the markup dispersion. Also, the alternative way to bring pro-competitive effects of trade back into the model would be an introduction of fixed production and exporting costs, which are absent in the Melitz and Ottaviano (2008) model as well as in our extension of their work.

⁹In Melitz and Ottaviano (2008), a household maximizes

$$U = q_0^c + \alpha \int_{\Omega} q^c(i) di - \frac{1}{2} \gamma \int_{\Omega} q^c(i)^2 di - \frac{1}{2} \eta \left(\int_{\Omega} q^c(i) di \right)^2 \quad s.t. \quad p_0^c q_0^c + \int_{\Omega} p(i) q^c(i) di = w,$$

where q_0^c is a consumption of the outside good.

where Ω is the set of all available differentiated good varieties. The degree of product differentiation between varieties in the utility function above is characterized by γ : with lower γ varieties become closer substitutes, and in the limit case of $\gamma = 0$, consumers care only about the total amount they consume. From the F.O.C., we get

$$\lambda p(i) = \alpha - \gamma q^c(i) - \eta \int_{\Omega} q^c(i) di, \quad (1)$$

where λ is a Lagrangian multiplier. Denote the aggregate quantity of all varieties consumed by an individual at Home, $\int_{\Omega} q^c(i) di$, by Q . Then

$$\lambda = \left(\alpha Q - \gamma \int_{\Omega} (q^c(i))^2 di - \eta Q^2 \right) / w. \quad (2)$$

Using the same logic as in Melitz and Ottaviano (2008), it can be shown that the set Ω^c of all varieties that are consumed ($q_i^c > 0$) is the largest subset of Ω that satisfies:

$$p_i \leq \frac{1}{\eta M + \gamma} \left(\frac{1}{\lambda} \gamma \alpha + \eta M \bar{p} \right) \equiv p_{\max},$$

where M is the measure of consumed varieties in Ω^c and $p_{\max} = (1/M) \int_{i \in \Omega^c} p_i di$ represents the choke price. Similarly, in the Foreign country

$$\lambda^* p(i) = \alpha - \gamma q^{c,*}(i) - \eta Q^*, \quad (3)$$

$$\lambda^* = \alpha Q^* - \gamma \int_{\Omega^*} (q^{c,*}(i))^2 di - \eta (Q^*)^2, \quad (4)$$

where $Q^* = \int_{\Omega^*} q^c(i) di$ is the aggregate quantity of all varieties consumed by a Foreign individual.

2.1.2 Production and Firm Behavior

Domestic Market. A firm with marginal cost MC sells its variety i to L consumers. Hence, it sells $q(i) = Lq^c(i)$ and maximizes its profit, $\pi(i) = pq(i) - MC(i)q(i)$, by choosing the appropriate level of $p(i)$. Given (1), we get

$$q^c(i) = \frac{1}{\gamma} \left(\alpha - \lambda p(i) - \eta \int_{\Omega} q^c(i) di \right),$$

so that the F.O.C. results in $q(i) = Lq^c(i) = L\lambda(p - MC(i))/\gamma$. There is a continuum of domestic firms at Home that derive their unit labor costs from the Pareto cost distribution given by $G(c) = \left(\frac{c}{c_M}\right)^k$, $c \in [0, c_M]$, $k > 1$, so that $MC(c) = wc$. Only firms with positive demand will sell domestically. Define cutoff c_D such that $q(c_D) = 0$ or, given wage w ,

$$p(c_D) = wc_D, \quad (5)$$

so that only firms with $c \leq c_D$ sell at Home. Using (1), we have

$$w\lambda c_D = \alpha - \eta Q. \quad (6)$$

Similarly, we can define the cutoff for Foreign domestic sellers, c_D^* (their initial cost distribution is $G^*(c) = \left(\frac{c}{c_M^*}\right)^k$, $c \in [0, c_M^*]$):

$$p(c_D^*) = c_D^*, \quad \lambda^* c_D^* = \alpha - \eta Q^*. \quad (7)$$

Given these cutoffs and the fact that $\lambda p(i) = \alpha - \gamma q^c(i) - \eta \int_{\Omega} q^c(i) di = w\lambda c_D - \gamma \frac{q(i)}{L}$, we can rewrite the expressions for the Home and Foreign domestic sellers as:

$$\begin{aligned} p(c) &= \frac{1}{2} w(c_D + c); & p^*(c^*) &= \frac{1}{2} (c_D^* + c^*), \\ q(c) &= \frac{L}{2\gamma} w\lambda(c_D - c); & q^*(c^*) &= \frac{L^*}{2\gamma} \lambda^* (c_D^* - c^*), \\ r(c) &= \frac{L}{4\gamma} w^2 \lambda \left((c_D)^2 - c^2 \right); & r^*(c^*) &= \frac{L^*}{4\gamma} \lambda^* \left((c_D^*)^2 - (c^*)^2 \right), \\ \pi(c) &= \frac{L}{4\gamma} w^2 \lambda (c_D - c)^2; & \pi^*(c^*) &= \frac{L^*}{4\gamma} \lambda^* (c_D^* - c^*)^2, \end{aligned} \quad (8)$$

where the last two lines above represent revenues and profits, respectively. Note that as in Melitz and Ottaviano (2008), a firm with a higher productivity (a lower cost) charges a lower price, makes higher sales, and earns higher profits. Moreover, more productive firms charge higher markups (a markup, $m(c) \equiv p(c)/MC(c)$, rises with c). This gives a rise to misallocation distortion, since more productive firms end up selling too little, while high cost producers tend to oversupply.

Exporting. The Foreign demand for the Home firm's variety is given by

$$\lambda^* p_X(i) = \alpha - \gamma q_X^c(i) - \eta Q^*, \quad (9)$$

while for the exporting Foreign firms we have

$$\lambda p_X^*(i) = \alpha - \gamma q_X^{c^*}(i) - \eta Q. \quad (10)$$

Then, given that Home and Foreign exporters face additional iceberg transportation costs of τ and τ^* , respectively, and using the same logic as before, we get

$$\begin{aligned} p_X(c) &= \frac{1}{2} w\tau(c_X + c); & p_X^*(c^*) &= \frac{1}{2} \tau^* (c_X^* + c^*), \\ q_X(c) &= \frac{L^*}{2\gamma} w\lambda^* \tau (c_X - c); & q_X^*(c^*) &= \frac{L}{2\gamma} \lambda \tau^* (c_X^* - c^*), \\ r_X(c) &= \frac{L^*}{4\gamma} w^2 \tau^2 \lambda^* \left((c_X)^2 - c^2 \right); & r_X^*(c^*) &= \frac{L}{4\gamma} \lambda (\tau^*)^2 \left((c_X^*)^2 - (c^*)^2 \right), \\ \pi_X(c) &= \frac{L^*}{4\gamma} w^2 \tau^2 \lambda^* (c_X - c)^2; & \pi_X^*(c^*) &= \frac{L}{4\gamma} \lambda (\tau^*)^2 (c_X^* - c^*)^2, \end{aligned} \quad (11)$$

where c_X and c_X^* are the cost cutoffs for Home and Foreign exporters, respectively, determined from $q_X(c_X) = 0$ and $q_X^*(c_X^*) = 0$. It can be shown directly that

$$c_X = \frac{c_D^*}{\tau w} \quad \text{and} \quad c_X^* = \frac{w c_D}{\tau^*}. \quad (12)$$

2.1.3 Equilibrium Conditions

The free entry condition implies that the expected profits from entering the market should be equal to the entry cost. Given the assumption of the Pareto cost distribution and (12), we can rewrite this condition for Home firms as

$$(FE) : \quad (c_M)^{-k} w \left[L \lambda (c_D)^{k+2} + \lambda^* L^* w^{-k-2} \tau^{-k} (c_D^*)^{k+2} \right] = Const_1, \quad (13)$$

where

$$Const_1 = 2\gamma f_e(k+1)(k+2). \quad (14)$$

Similarly, the free entry condition for Foreign firms is

$$(FE)^* : \quad (c_M^*)^{-k} \left[L^* \lambda^* (c_D^*)^{k+2} + \lambda L w^{k+2} (\tau^*)^{-k} (c_D)^{k+2} \right] = Const_1. \quad (15)$$

Next, let us look at the mass of active firms at Home, M . Due to free entry, total profits in the economy are zero, i.e., total revenues are equal to the labor payment:

$$R = wL, \quad \text{where} \quad R = M \left(\bar{r}_D + \frac{G(c_X)}{G(c_D)} \bar{r}_X \right),$$

where $\bar{r}_D$ and $\bar{r}_X$ are the expected revenues from domestic and export sales conditional on getting a cost draw below the corresponding cutoff, given by

$$\bar{r}_D = \int_0^{c_D} r(c) \frac{dG(c)}{G(c_D)} = \frac{1}{2\gamma} L w^2 \lambda \frac{(c_D)^2}{k+2}, \quad \bar{r}_X = \int_0^{c_X} r_X(c) \frac{dG(c)}{G(c_X)} = \frac{1}{2\gamma} L^* w^2 \lambda^* \tau^2 \frac{(c_X)^2}{k+2}. \quad (16)$$

From the (FE) condition, $G(c_D) \bar{r}_D + G(c_X) \bar{r}_X = w f_e(k+1)$, so that

$$M = \frac{L}{f_e(k+1)} \left(\frac{c_D}{c_M} \right)^k.$$

Similarly, the mass of active Foreign firms is

$$M^* = \frac{L^*}{f_e(k+1)} \left(\frac{c_D^*}{c_M^*} \right)^k. \quad (17)$$

Then the masses of entrants in each economy can be calculated as

$$M_e = G(c_D) M = \frac{L}{f_e(k+1)}, \quad M_e^* = G^*(c_D^*) M^* = \frac{L^*}{f_e(k+1)}. \quad (18)$$

Now let us derive the trade balance condition. It is given by $M_X \bar{r}_X = M_X^* \bar{r}_X^*$, where $M_X = (G(c_X)/G(c_D))M$ and $M_X^* = (G^*(c_X^*)/G^*(c_D^*))M^*$ are the masses of Home and Foreign exporters, respectively. Then, by using (12) and (18), we get

$$(TB) : \quad \lambda^* (\tau^*)^k (c_D^*)^{k+2} (c_M^*)^k = w^{2k+2} \lambda \tau^k (c_D)^{k+2} (c_M)^k. \quad (19)$$

Finally, we need to derive the equations for λ and λ^* . As shown in Appendix A, they can be written as

$$(\lambda) : \quad w\lambda = \frac{\alpha}{c_D} - \frac{k+2}{k+1} \eta \frac{1}{(c_D)^2}, \quad (\lambda)^* : \quad \lambda^* = \frac{\alpha}{c_D^*} - \frac{k+2}{k+1} \eta \frac{1}{(c_D^*)^2}. \quad (20)$$

We can use these expressions to exclude λ and λ^* from the set of equilibrium variables, so that in the equilibrium, we have 3 unknown variables, w , c_D , and c_D^* , that can be found from 3 conditions below:¹⁰

$$\begin{aligned} (FE) : \quad & \left(\alpha c_D - \frac{k+2}{k+1} \eta \right) (c_D)^k \left[L c_M^{-k} + L^* (c_M^*)^{-k} w^k (\tau^*)^{-k} \right] = Const_1, \\ (FE)^* : \quad & \left(\alpha c_D^* - \frac{k+2}{k+1} \eta \right) (c_D^*)^k \left[L^* (c_M^*)^{-k} + L c_M^{-k} w^{-k} \tau^{-k} \right] = Const_1, \\ (TB) : \quad & L (c_M)^{-k} + L^* (c_M^*)^{-k} w^k (\tau^*)^{-k} = w^{2k+1} \left(\frac{\tau c_M}{\tau^* c_M^*} \right)^k \left[L^* (c_M^*)^{-k} + L (c_M)^{-k} w^{-k} \tau^{-k} \right]. \end{aligned}$$

2.1.4 Welfare

As shown in Appendix A, welfare per capita at Home can be written as a function of a cost cutoff for domestic producers only:

$$U = \frac{1}{2} \frac{2k+3}{k+1} \left[\frac{\alpha}{c_D} - \eta \frac{k+2}{2k+3} \frac{1}{(c_D)^2} \right]. \quad (21)$$

It implies that welfare at Home rises with lower cost (higher productivity) of a marginal local seller. Similarly, welfare per capita in the Foreign country has the same formula as (21) with c_D^* instead of c_D .

2.2 Small Economy Case

In this Section we discuss the case of two countries, Home and Foreign, with Home being a “small economy” relative to the Foreign one. As in Demidova and Rodríguez-Clare (2009), this involves 3

¹⁰Note that the (TB) condition above implies that we can solve it for the Home wage, and then all the other variables in the equilibrium can be calculated as functions of wage.

assumptions: (i) the Foreign demand for Home varieties depends only on their prices, i.e., aggregate variables in the Foreign demand function are not affected by Home; (ii) the cost distribution of Foreign producers is fixed; and (iii) the mass of available Foreign varieties is fixed (the last two assumptions mean that the mass of Foreign firms and Foreign wage are not affected by changes at Home).¹¹

The equilibrium derivations are similar to those in the case of two large economies with a few modifications discussed below. First, note that the derivations for Home firms selling locally do not change. Next, let us look at exporting.

Home Exporting. “Small economy” assumption (i) implies that the Foreign demand for the Home firm’s variety is given by

$$q_X(i) = A - Bp_X(i), \quad (22)$$

where A and B are fixed constants. From the profit maximization problem of a Home exporter, we have

$$p_X(c) = \frac{A}{2B} + \frac{MC_X(c)}{2} \quad \text{and} \quad q_X(c) = \frac{1}{2}(A - B(MC_X(c))), \quad (23)$$

where the marginal cost of an exporter is $MC_X(c) = w\tau c$. Then, we can define the cost cutoff of a marginal exporter from $q_X(c_X) = 0$,

$$c_X = \frac{A}{B} \frac{1}{w\tau}. \quad (24)$$

By using the same logic as before, we get the following expressions:

$$\begin{aligned} p_X(c) &= \frac{1}{2}w\tau(c_X + c); & q_X(c) &= Bw\tau(c_X - c); \\ r_X(c) &= \frac{1}{4}Bw^2\tau^2((c_X)^2 - c^2); & \pi_X(c) &= \frac{1}{4}w^2\tau^2B(c_X - c)^2. \end{aligned}$$

Foreign Exporting. “Small economy” assumption (ii) implies that *active* Foreign firms have the following cost distribution: $G(c) = \left(\frac{c}{c_M^*}\right)^k$, $k > 1$, $c \in [0, c_M^*]$, which is not affected by changes at Home. Finally, “small economy” assumption (iii) states that the mass of active Foreign firms, M^* , is fixed, i.e., entry abroad is not not affected by changes at Home as well. We normalize M^* to 1. Note that not all active Foreign firms sell their goods in the Home market: only firms with $c \leq c_X^*$ become exporters, where, as before, $c_X^* = w\tau c_D/\tau^*$.

Equilibrium Conditions. The new free entry condition for Home firms can be written as

$$(FE): \quad (c_M)^{-k} \left[Lw\lambda(c_D)^{k+2} + \gamma B \left(\frac{A}{B}\right)^{k+2} \tau^{-k} w^{-k-1} \right] = Const_1, \quad (25)$$

¹¹See Appendix B for justification of these three assumptions in our model.

where as before, $Const_1 = 2\gamma f_e (k+1)(k+2)$. Also, it is straightforward to show that, as before, M_e is given by (18). The new trade balance condition is given by

$$(TB) : \quad w^{2k+2} \lambda (c_D)^{k+2} \left(\frac{\tau c_M}{\tau^* c_M^*} \right)^k = Const_2, \quad (26)$$

where

$$Const_2 = \gamma B \left(\frac{A}{B} \right)^{k+2} \frac{1}{f_e (k+1)}. \quad (27)$$

Finally, using the same logic as before, we can derive the expression for λ , which is still given by (20), and exclude it from the set of the equilibrium variables, so that we end up with only 2 equations for 2 unknown variables in the equilibrium, w and c_D :

$$(FE) : \quad (c_M)^{-k} \left[L \left(\alpha c_D - \frac{k+2}{k+1} \eta \right) (c_D)^k + \gamma B \left(\frac{A}{B} \right)^{k+2} \tau^{-k} w^{-k-1} \right] = Const_1, \quad (28)$$

$$(TB) : \quad w^{2k+1} \left(\alpha c_D - \frac{k+2}{k+1} \eta \right) (c_D)^k \left(\frac{\tau c_M}{\tau^* c_M^*} \right)^k = Const_2. \quad (29)$$

The formula for welfare per capita at Home is the same as before and is given by (21). This completes the description of the model of a small Home economy.

3 Falling Trade Costs

In this Section we show that in our setting that does not incorporate an outside good assumption, welfare of a liberalizing country rises as it unilaterally reduces trade barriers to foreign firms. Hence, the results of Melitz and Ottaviano (2008) are reversed.

Consider unilateral trade liberalization by Home in form of falling trade costs of Foreign exporters, i.e., τ^* falls. Note that another interpretation of a fall in τ^* in the literature is a fall in a wasteful tariff, since the analysis ignores tariff revenues and changes in them. We get the following result:

Lemma 1 *Whether Home is a small or large economy, the (FE) condition implies a positive relationship between c_D and w , while the (TB) condition implies a negative relationship between c_D and w .*

Proof. See Appendix C. ■

We can depict both relationships in Figure 1, where the FE and TB curves represent the (FE)

Figure 1: Unilateral Trade Liberalization by Home

and (TB) conditions, respectively.¹² Intuitively, the FE curve is upward sloping, since high wage deters entry, letting less efficient (high cost) firms survive. The TB curve is downward sloping, since, to keep trade balance, high wage at Home must be compensated by higher efficiency (lower cost) of Home firms. The intersection of two curves gives the unique equilibrium values of w and c_D . Moreover, it is straightforward to show that a reduction in inward variable trade barriers at Home, τ^* , affects only the TB curve by shifting it down, which immediately proves that both w and c_D fall as τ^* falls. Also, from the $(FE)^*$ condition in the case of a large Home economy, it can be shown directly that c_D^* falls with falling w . Finally, recall that welfare in both economies falls with the cost cutoff for local producers there. Thus, we proved that:

Proposition 1 *Whether Home is a small or large economy, unilateral trade liberalization by Home in form of falling per unit trade costs raises welfare there. Moreover, in the case of two large economies, welfare of the Foreign country rises as well.*

The comparison of Proposition 1 with the outcome in Melitz and Ottaviano (2008) shows that the outside good assumption incorporated in Melitz and Ottaviano (2008) is crucial for their

¹²Note that our Figure 1 for the case of variable markups resembles the one in Demidova and Rodríguez-Clare (2013) for CES preferences. This is not a coincidence. We relate w and the cost cutoff for domestic sellers, c_D , while their graphical analysis relates w and the productivity cutoff for exporters, φ_X . In the Melitz (2003) model the productivity cutoff for exporters is negatively related to the productivity cutoff for domestic sellers, which, in turn, is an inverse of the cost cutoff. Hence, we have similar relationships between w and c_D in our Figure 1 and w and φ_X in Figure 1 in Demidova and Rodríguez-Clare (2013).

results regarding unilateral trade liberalization: while UTL by Home is welfare-reducing for the Home government in their model, once the outside good assumption, and, in turn, the distortion that it creates, is dropped from the model, full trade liberalization becomes an optimal policy, whether Home is a small or large economy, raising welfare of all trading partners.

4 Non-Wasteful Import Tariffs

4.1 The Model

In this Section we consider a non-wasteful import tariff. In particular, assume that if a Foreign firm charges price p_X^* , then the Home government collects tariff revenues of $(t - 1)p_X^*/t$ per unit sold, so that the Foreign firm receives only p_X^*/t . The question is whether charging $t > 1$ (i.e., a strictly positive tariff) is a desirable policy for the Home government. In other words, we want to see how unilateral trade liberalization affects welfare at Home, when the policy instrument is an import tariff instead of per unit trade costs.

Our discussion begins with the derivations that are common for both the large and small Home economy cases. If the import tariff is non-wasteful, then the tariff revenues it generates, T , must be included into the Home national income. Hence, it becomes

$$I = wL + T.$$

Then, the income per capita becomes $I/L = w + (T/L)$ instead of just a labor payment, wage w . This means that all our derivations for Home consumers remain the same except for the new equation instead of (2):

$$(I/L) \lambda = \alpha Q - \gamma \int_{\Omega} (q^c(i))^2 di - \eta Q^2. \quad (30)$$

Next, all derivations for Home producers do not change, which gives us the same (FE) condition as before (see (13) for the large economy case and (28) for the small economy case). The main change for the Foreign exporters is that while their goods are sold at p_X^* in the Home market, i.e., the demand for their goods is defined from $\lambda(p_X^*) = \alpha - \gamma q_X^* - \eta Q$, the exporters collect only p_X/t so that their profits are $\pi_X = (p_X^*/t - MC_X^*) q_X^*$. This gives the following cost cutoff for the marginal Foreign exporter:

$$c_X^* = \frac{wc_D}{t\tau^*},$$

so that the price set by the Foreign exporter with cost draw c is

$$p_X^*(c) = \frac{1}{2}(wc_D + t\tau^*c) = \frac{1}{2}t\tau^* \left(\frac{wc_D}{t\tau^*} + c \right) = \frac{1}{2}t\tau^*(c_X^* + c).$$

All other expressions for Foreign exporters now include t as well :

$$q_X^*(c) = t \frac{L}{2\gamma} \lambda \tau^* (c_X^* - c), \quad r_X^*(c) = \frac{1}{t} p_X^*(c) q_X^*(c) = t \frac{L}{4\gamma} \lambda (\tau^*)^2 \left((c_X^*)^2 - (c)^2 \right), \quad \pi_X^*(c) = t \frac{L}{4\gamma} \lambda (\tau^*)^2 (c_X^* - c)^2$$

From now on we will look at the large and small Home economy cases separately.

Large Home Economy Case. Given the changes above, the $(FE)^*$ condition in the large Home economy case can be written as

$$(FE)^* : (c_M^*)^{-k} \left[L^* \lambda^* (c_D^*)^{k+2} + \lambda L \frac{w^{k+2}}{t^{k+1}} (\tau^*)^{-k} (c_D)^{k+2} \right] = Const_1,$$

where $Const_1$ is defined by (14). Next, to derive the expression for the mass of active firms at Home, let us look at the total expenditures at Home:

$$wL + T = t(M_x^* \bar{r}_x^*) + M \bar{r}_d, \quad \text{or} \quad wL + (t-1)(M_x^* \bar{r}_x^*) = t(M_x^* \bar{r}_x^*) + M \bar{r}_d.$$

The (TB) condition implies that $M_x \bar{r}_x = M_x^* \bar{r}_x^*$. Then

$$wL = M_x^* \bar{r}_x^* + M \bar{r}_d = M_x \bar{r}_x + M \bar{r}_d = R, \quad (31)$$

i.e., total revenues earned by Home firms are equal to labor payments at Home. This implies that our derivations for the masses of firms at Home and abroad do not change, and as before, M_e and M_e^* are given by (18).

Given the new formula for c_X^* , the (TB) condition can be written as

$$(TB) : \frac{w^{2k+2}}{t^{k+1}} \lambda (c_D)^{k+2} \left(\frac{\tau c_M}{\tau^* c_M^*} \right)^k = \lambda^* (c_D^*)^{k+2}. \quad (32)$$

The derivations for λ^* do not change: λ^* is still given by (20). However, compared to (40) in Appendix A (where $I/L = w$), now we have:

$$\lambda = \frac{\gamma \int_{\Omega} (q^c(i))^2 di}{w c_D Q - I/L},$$

where $I/L \neq w$. Using the same logic as in Appendix A,¹³ we get $Q = \frac{I}{wL} \frac{k+2}{k+1}$ and

$$(\lambda) : w \lambda c_D = \left(\alpha - \eta \frac{k+2}{k+1} * \frac{1}{c_D} \right) / \left(1 + \eta \frac{k+2}{k+1} y \right), \quad (33)$$

¹³The only new derivation needed is for the national income per capita: $\frac{I}{wL} = 1 + \frac{(t-1)}{wL} M_x^* \bar{r}_x^* = 1 + M_e^* \frac{(t-1)}{wL} \left[t \frac{L}{2\gamma} \lambda \frac{(\tau^*)^2}{k+2} \frac{(c_X^*)^{k+2}}{(c_M^*)^k} \right] = 1 + t M_e^* \frac{(t-1)}{2\gamma(k+2)t^{k+2}} \left(\frac{w c_D}{c_M \tau^*} \right)^k c_D (w \lambda c_D) = 1 + y c_D (\alpha - \eta Q) = 1 + y \alpha c_D - y \eta \frac{I}{wL} \frac{k+2}{k+1}$, so that $I/wL = (1 + \alpha c_D y) / (1 + \eta \frac{k+2}{k+1} y)$.

where

$$y = \frac{t-1}{2\gamma(k+2)t^{k+1}} \left(\frac{wc_D}{\tau^*c_M} \right)^k M_e^*. \quad (34)$$

Note that if $t = 1$, then $y = 0$ so that we get the same equations as in the case of a wasteful tariff in Section 3. Using (33) to exclude λ , we end up with 3 unknown variables, w , c_D , and c_D^* , and 3 equilibrium conditions, (FE) , $(FE)^*$, and (TB) .

Small Home Economy Case. As before, we do not have the $(FE)^*$ condition in the small Home economy case, since Foreign entry is fixed. Also, using the same logic as in the case of the large Home economy, it can be shown that the expressions for the masses of firms at Home do not change and given by (18). Next, by using the new expression for c_X^* , we can re-write the (TB) condition as

$$\text{Small Economy case: } (TB) : \frac{w^{2k+2}}{t^{k+1}} \lambda (c_D)^{k+2} \left(\frac{\tau c_M}{\tau^* c_M^*} \right)^k = Const_2, \quad (35)$$

where $Const_2$ is defined by (27). Finally, we can derive the new expression for λ , which is again given by (33) with the main difference being in the definition of y , where now $M_e^* = 1$.

Welfare. In both cases of the large and small Home economy, the expression for welfare of the Foreign country does not change, but the expression for welfare at Home becomes more complicated. See Appendix D for more details on it.

4.2 Optimal Non-Wasteful Import Tariffs

We leave all the proofs for the cases of large and small Home economies to Appendices D and E, respectively. Here we provide the main results only. First, we have:

Proposition 2 *In the case of a small Home economy, there exists a unique import tariff that maximizes welfare there:*

$$t^{opt} = 1 + \frac{1}{k}, \quad (36)$$

where k is the shape parameter of the Pareto cost distribution.

This result is in stark contrast to Proposition 1: full trade liberalization is no longer a preferable policy for the Home government even in the absence of the outside good sector, which means that whatever the distortions created by variable markups in addition to the markup and consumption-surplus distortions present in the CES case, protection is needed to deal with them. We will discuss

the role of variable markups in Section 5. Meanwhile, note that the level of protection depends on the degree of firm heterogeneity: lower k (higher firm heterogeneity) results in higher need for protection, and when $k \rightarrow \infty$, i.e., all firms become identical, protection is no longer needed. The intuition is that if firm dispersion is high, then even without a tariff, only few low-cost firms survive. This has two effects on the desired policy: first, low-cost firms charge higher markups so that the markup distortion at Home gets worse. Second, since only few Foreign firms survive, the Home loss in imported variety due to the tariff becomes smaller. As a result, markup distortion outweighs the consumption-surplus one even more, so higher protection is needed.

Now let us turn to the large Home economy case. We get the following result:

Proposition 3 *In the case of a large Home economy, there exists a unique import tariff that maximizes welfare there. It can be found as a solution of the following equation:*

$$t^{opt} = 1 + \frac{1}{k} + \frac{1}{k} \frac{L}{L^*} \left(\frac{c_M^*}{c_M \tau} \right)^k w^{-k}, \quad (37)$$

where w is wage at Home.

The comparison of (36) and (37) reveals that the level of protection is higher for the large Home economy, which is not surprising due to the terms of trade externality that is captured by the last term: it depends on the relative market size, L/L^* , the relative productivity, $(c_M^*/c_M)^k$, and openness of the Home economy, τ^{-k} . Moreover, as $L/L^* \rightarrow 0$ (Home becomes the small economy), (37) converges to (36). Note that this result strongly resembles the one derived by Felbermayr, Jung and Larch (2013) for the case of constant markups, and as in their case, the analysis of the properties of t^{opt} is complicated by the endogenous nature of wage w .¹⁴

The difference between two types of trade barriers is also emphasized by Bagwell and Lee (2015), who also show in the model of two symmetric countries that unlike an increase in trade costs studied in Melitz and Ottaviano (2008), a small increase in the import tariff imposed by the Home economy raises welfare there, while reducing welfare abroad. Moreover, such a change increases the average price at Home, while reducing that abroad, giving a rise to a Metzler paradox in their model. However, the fact that the Melitz and Ottaviano (2008) model generates this paradox depends crucially on the assumption of the outside good. As we show in Appendix E, once this assumption is dropped, both Home and Foreign average prices rise so that a Metzler paradox disappears: both the Home and Foreign prices increase with a higher import tariff.

¹⁴See (55) that implicitly defines w as a function of t , which allows for the comparative statics analysis of t^{opt} .

Given such similarity of the results in the models with constant and variable markups, the natural question then is what role variable markups play. We answer it in the next Section.

5 Unilateral Trade Liberalization and Markups

To begin, let us discuss a reduction in per unit trade costs. Even though the average local markup as well as the dispersion of local markups remain unchanged as τ^* falls, this does not mean that variable markups play no role as countries liberalize. In particular, as Edmond, Midrigan and Xu (forthcoming) and Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2012) point out, tracking down the changes in the local markup distribution alone is not enough. First, Edmond, Midrigan and Xu (forthcoming) argue that higher markups on imported goods can outweigh the reduction in local markups and make the misallocation distortion worse. To study this effect, they look at the revenue-weighted harmonic average of markups. In our case, however, a unilateral fall in trade costs of Foreign exporters does not affect the average markup, again, due to the assumption of Pareto cost distribution. Second, Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2012) show that variable markups create a new source of gains or losses from trade liberalization, depending on whether low cost firms, which charge high markups and under-supply their varieties, end up growing in size or not. In this case the effect of trade liberalization of labor reallocation on welfare of country j depends on the sign of the covariance of the markup, $m(w, i)$ charged by a firm in country j that produces variety w for market i , and a change in its labor share that is needed to produce this variety for market i :

$$\text{cov} \left(m(\omega, i), \frac{dl(\omega, i)}{L_j} \right) = \sum_i \int_{\omega \in \Omega_{ji}} \left[m(\omega, i) \frac{dl(\omega, i)}{L_j} \right] d\omega, \quad (38)$$

where $l(\omega, i)$ is the total employment associated with a production of variety ω in country j for sales in country i ,¹⁵ and Ω_{ji} is the set of all varieties produced in country j for country i . In other words, trade liberalization has a positive (negative) effect on welfare in country j , if this covariance is positive (negative). The important property of the formula above is that it depends not only on the firms' decisions in their local market, but also on their exporting decisions. In our model, the

¹⁵Note that (38) is obtained due to the fact that for any labor re-allocations, the size of the economy remains the same so that $\sum_i \int_{\omega \in \Omega_{ji}} \left[\frac{dl(\omega, i)}{L_j} \right] d\omega = 0$.

formula above can be rewritten as

$$\begin{aligned} cov\left(m(\omega, i), \frac{dl(\omega, i)}{L_j}\right) &= M \int_0^{c_D} \frac{p(c)}{wc} \frac{d[cq(c)]}{L} \frac{dG(c)}{G(c_D)} + M_x \int_0^{c_x} \frac{p_X(c)}{w\tau c} \frac{d[\tau cq_X(c)]}{L} \frac{dG(c)}{G(c_X)} \\ &= \frac{M}{wL} \int_0^{c_D} p(c) d[q(c)] \frac{dG(c)}{G(c_D)} + \frac{M_x}{wL} \int_0^{c_x} p_X(c) d[q_X(c)] \frac{dG(c)}{G(c_X)} \end{aligned} \quad (39)$$

As we show in Appendix F, for a small fall in τ^* , the expression above is negative. Hence, although unilateral trade liberalization is beneficial for the Home economy as a whole, its gains are mitigated by misallocation distortion that gets worse as τ^* starts to fall.¹⁶ In other words, while qualitatively our UTL results are the same as in the case of the Melitz (2003) model with CES preferences (see Demidova and Rodriguez-Clare, 2013), in the case of variable markups welfare gains seem to be smaller.

Now let us turn to import tariffs. Given our discussion above, one may expect that the level of optimal tariffs in our model should be higher compared to that in the Melitz (2003) model with constant markups. This is exactly the case. In the small economy case, the direct comparison of the optimal tariff derived in Section 4, $t^{opt} = 1 + \frac{1}{k}$, with that derived by Demidova and Rodríguez-Clare (2009) for the Melitz (2003) model with Pareto cost distribution, $t = 1 + \frac{\sigma-1}{k\sigma-(\sigma-1)}$, where σ is the elasticity of substitution from the CES preferences, shows that in the first case the optimal tariff is higher. With some caution, a similar comparison can be made in the large economy case, where we can look at the tariff derived in Section 4 and the one derived by Felbermayr, Jung and Larch (2013) for the Melitz (2003) model with Pareto cost distribution.

Another point we want to make in this Section is that there is an important difference between a fall in per unit trade costs and an import tariff. Unlike movements in trade costs that leave the aggregate markup unchanged, a non-wasteful import tariff has ability to affect it. To avoid analytical difficulties, let us simplify the large economy case considered in Section 4 by imposing some symmetry on two countries, i.e., assume that $L = L^*$, $c_M = c_M^*$, $\tau = \tau^* = 1$. Then the aggregate markup becomes

$$\bar{m} = \frac{M}{M + M_X^*} \int_0^{c_D} m(c) \frac{dG(c)}{G(c_D)} + \frac{M_X^*}{M + M_X^*} \int_0^{c_D} m_X^*(c) \frac{dG(c)}{G(c_X^*)} = \frac{1}{2} \frac{2k-1}{k-1} \frac{M + tM_X^*}{M + M_X^*}.$$

As we show in Appendix F, when an import tariff t starts to rise from 1, the average markup falls. In other words, trade liberalization by Home in form of a falling import tariff, when the level of protection is small already, actually increases the average markup faced by consumers there, which, according to Edmond, Midrigan and Xu (forthcoming), implies the negative pro-competitive effect.

¹⁶Due to complexity of the analysis of large falls in τ^* , we leave it to future work.

Hence, one has to be careful while extending the results derived for the trade costs to the analysis of import tariffs.

6 Conclusion

In this paper we have studied the implications of trade cost reductions and import tariffs in the extension of the Melitz and Ottaviano (2008) model without the outside good. Our conclusions can be broadly summarized as follows. First, we find that in contrast to Melitz and Ottaviano (2008), a reduction in per unit trade costs raises, and not reduces, welfare of the liberalizing country as well as welfare of its trading partner. Thus, the breakdown of optimality of laissez-faire in Melitz and Ottaviano (2008) can be explained by the distortion created by the presence of the outside good sector. Second, we derive the optimal values of import tariffs for the large and small Home economies and show that as in the models with monopolistic competition and CES preferences, protection is always a desirable policy for the Home government. The main difference between the policy implications from the CES models and our setting is that the level of protection in our case is higher due to the negative pro-competitive effect caused by variable markups.

Given the obtained results, there are several potential avenues for future research. First, as discussed in Introduction, it is possible that the absence of pro-competitive gains from trade in our model can be explained by the use of another popular assumption in literature on firm heterogeneity, namely, cost distributions being specified as Pareto. Thus, the natural question is how the deviation from this assumption will affect our conclusions. Second, following Melitz and Ottaviano (2008), we have relied on a very specific demand structure. It would be interesting to see whether the impact of unilateral trade liberalization changes if one instead uses other types of preferences that generate variable markups, e.g., variable elasticity of substitution (VES) preferences as in Dhingra and Morrow (2012) and Zhelobodko et al. (2012) or the quadratic mean of order r (QMOR) expenditure function as in Feenstra (2014). Third, we have left Nash trade policies out of the scope of this paper.¹⁷ The question is then if protection remains the optimal policy when all trading countries, and not just the Home economy, have the ability to choose their tariffs. We leave all these questions to future work.

¹⁷See Bagwell and Lee (2015) for the discussion of Nash trade policies in the Melitz and Ottaviano (2008) model with the outside good.

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Appendix

Appendix A: Equilibrium Conditions in Case of 2 Large Economies

Derivations for Lagrangian Multipliers

From (6), $\lambda p q^c = (\alpha - \eta Q - \gamma q^c) q^c = (\lambda w c_D - \gamma q^c) q^c$. By integrating both parts over all varieties sold at Home, we get

$$\lambda \int_{\Omega} p(i) q^c(i) di = w\lambda = \lambda w c_D Q - \gamma \int_{\Omega} (q^c(i))^2 di,$$

so that

$$\lambda = \frac{\gamma \int_{\Omega} (q^c(i))^2 di}{w(c_D Q - 1)}, \quad (40)$$

where from (12), we get

$$\begin{aligned}
Q &= M \int_0^{c_D} q^c(c) \frac{dG(c)}{G(c_D)} + M_X^* \int_0^{c_X^*} q_X^{c,*}(c) \frac{dG^*(c)}{G^*(c_X^*)} \\
&= \frac{1}{2\gamma} \frac{1}{k+1} \lambda (c_D)^{k+1} \left[M_e w (c_M)^{-k} + M_e^* (c_M^*)^{-k} (\tau^*)^{-k} w^{k+1} \right], \\
\int_{\Omega} (q^c(i))^2 di &= M \int_0^{c_D} (q(c))^2 \frac{dG(c)}{G(c_D)} + M_X^* \int_0^{c_X^*} (q_X^{c,*}(c))^2 \frac{dG^*(c)}{G^*(c_X^*)} \\
&= \frac{1}{2\gamma^2} \frac{(c_D)^{k+2} w \lambda^2}{(k+1)(k+2)} \left[M_e w (c_M)^{-k} + M_e^* (c_M^*)^{-k} (\tau^*)^{-k} w^{k+1} \right] = \frac{1}{\gamma} w \lambda c_D \frac{1}{k+2} Q.
\end{aligned}$$

Thus, (40) can be re-written as $\lambda w (c_D Q - 1) = w \lambda c_D \frac{1}{k+2} Q$, so that

$$Q = \frac{k+2}{k+1} \frac{1}{c_D}. \quad (41)$$

Then from (2),

$$w \lambda = \alpha Q - \gamma \int_{\Omega} (q^c(i))^2 di - \eta Q^2 = \alpha \frac{k+2}{k+1} \frac{1}{c_D} - w \lambda \frac{1}{k+1} - \eta \left(\frac{k+2}{k+1} \frac{1}{c_D} \right)^2,$$

which we can solve for $w \lambda$:

$$(\lambda): \quad w \lambda = \frac{\alpha}{c_D} - \frac{k+2}{k+1} \eta \frac{1}{(c_D)^2}. \quad (42)$$

Similarly,

$$Q^* = \frac{k+2}{k+1} \frac{1}{c_D^*} \quad \text{and} \quad (\lambda^*): \quad \lambda^* = \frac{\alpha}{c_D^*} - \frac{k+2}{k+1} \eta \frac{1}{(c_D^*)^2}. \quad (43)$$

Welfare

From (41), per capita welfare at Home is given by

$$U = \frac{k+1}{2(k+2)} Q \left[\frac{\alpha(2k+3)}{k+1} - \eta Q \right] = \frac{1}{2} \frac{2k+3}{k+1} \left[\frac{\alpha}{c_D} - \eta \frac{k+2}{2k+3} \frac{1}{(c_D)^2} \right]. \quad (44)$$

Per capita welfare in the Foreign country can be derived in a similar manner.

Appendix B: Justification of Small Economy Assumption

Let us look at the equilibrium in the case of two countries considered in Section 2:

$$\begin{aligned}
(FE) \quad &: \quad w \left[L\lambda (c_D)^{k+2} + \lambda^* L^* w^{-k-2} \tau^{-k} (c_D^*)^{k+2} \right] = Const_1 (c_M)^k, \\
(FE)^* \quad &: \quad L^* \lambda^* (c_D^*)^{k+2} + \lambda L w^{k+2} (\tau^*)^{-k} (c_D)^{k+2} = Const_1 (c_M^*)^k, \\
(\lambda) \quad &: \quad w\lambda = \frac{\alpha}{c_D} - \frac{k+2}{k+1} \eta \frac{1}{(c_D)^2}, \\
(\lambda)^* \quad &: \quad \lambda^* = \frac{\alpha}{c_D^*} - \frac{k+2}{k+1} \eta \frac{1}{(c_D^*)^2}, \\
(TB) \quad &: \quad \lambda^* (\tau^* c_M^*)^k (c_D^*)^{k+2} = w^{2k+2} \lambda (\tau c_M)^k (c_D)^{k+2}.
\end{aligned}$$

When L/L^* goes to 0, i.e., when Home becomes small relative to the Foreign country, we have

$$(FE)^*: \quad \lambda^* (c_D^*)^{k+2} + \lambda \frac{L}{L^*} w^{k+2} (\tau^*)^{-k} (c_D)^{k+2} = \frac{Const_1 (c_M^*)^k}{L^*},$$

or $\lambda^* (c_D^*)^{k+2} \approx Const_1 (c_M^*)^k / L^*$. In other words, $\lambda^* (c_D^*)^{k+2}$ is no longer affected by changes in Home's variables. Then from (λ^*) , c_D^* and M_t^* are not affected as well, which justifies assumptions (ii) and (iii) and equation (29). We are left with 3 unknowns now, w , c_D , and λ , and 3 equations,

$$\begin{aligned}
(FE) \quad &: \quad wL\lambda (c_D)^{k+2} + \left(\lambda^* L^* (c_D^*)^{k+2} \right) w^{-k-1} \tau^{-k} = Const_1 (c_M)^k, \\
(\lambda) \quad &: \quad w\lambda = \frac{\alpha}{c_D} - \frac{k+2}{k+1} \eta \frac{1}{(c_D)^2}, \\
(TB) \quad &: \quad w^{2k+2} \lambda \left(\frac{\tau c_M}{\tau^* c_M^*} \right)^k (c_D)^{k+2} = \frac{Const_1 (c_M^*)^k}{L^*}.
\end{aligned}$$

Note that the demand for Home variety abroad is given by

$$q_X(i) = L^* x_X(i) = \frac{L^*}{\gamma} [\alpha - \eta X^* - \lambda^* p_X(i)] = \frac{L^*}{\gamma} [\lambda^* c_D^* - \lambda^* p_X(i)],$$

where $\frac{L^*}{\gamma} \lambda^* c_D^*$ and $\frac{L^*}{\gamma} \lambda^*$ are now constants. Let us denote them by A and B , respectively, so that

$$q_X(i) = A - B p_X(i),$$

as implied by assumption (i). Then we can rewrite (FE) as

$$\begin{aligned}
(FE) \quad &: \quad wL\lambda (c_D)^{k+2} + \lambda^* L^* w^{-k-1} \tau^{-k} (c_D^*)^{k+2} = Const_1 (c_M)^k, \quad \text{or} \\
(FE) \quad &: \quad Lw\lambda (c_D)^{k+2} + \gamma B \left(\frac{A}{B} \right)^{k+2} \tau^{-k} w^{-k-1} = Const_1 (c_M)^k,
\end{aligned}$$

which is exactly what we have as the (FE) condition given by (28). Thus, we proved that our “small economy” assumptions follow directly from the case of two economies, when one of them becomes much smaller relative to the other one.

Appendix C: Proof of Lemma 1

Let us first look at the case of 2 large economies. Using the (FE) and (FE)*, we can re-write the (TB) condition as

$$w^{2k+1} \left(\alpha c_D - \frac{k+2}{k+1} \eta \right) (c_D)^k \left(\frac{\tau c_M}{\tau^* c_M^*} \right)^k = \frac{Const_1 (c_M^*)^k}{L^* (c_M^*)^{-k} + L c_M^{-k} w^{-k} \tau^{-k}}.$$

In the case of a small economy (see (29)) the RHS of the equation above becomes a constant, which implies the negative relationship between w and c_D . We can prove that this is also true in the case of 2 large economies by re-writing the condition above as

$$w^{k+1} \left[L^* (c_M^*)^{-k} w^k + L c_M^{-k} \tau^{-k} \right] \left(\alpha c_D - \frac{k+2}{k+1} \eta \right) (c_D)^k \left(\frac{\tau c_M}{\tau^* c_M^*} \right)^k = Const_1 (c_M^*)^k.$$

Hence, in the case of 2 large economies, from the (TB) condition, w and c_D are negatively related. Next, by using (λ) and (λ^*) , we can re-write the (FE) condition given by (13) as

$$(c_M)^{-k} \left[L \left(\alpha c_D - \frac{k+2}{k+1} \eta \right) (c_D)^k + w^{-k-1} \tau^{-k} L^* \left(\alpha c_D^* - \frac{k+2}{k+1} \eta \right) (c_D^*)^k \right] = Const_1.$$

In the case of a small economy (see (28)) term $L^* \left(\alpha c_D^* - \frac{k+2}{k+1} \eta \right) (c_D^*)^k$ becomes a constant, which implies the positive relationship between w and c_D . We can prove the same result for the case of 2 large economies, if we use the (FE)* condition and re-write the equation above as

$$(c_M)^{-k} \left[L \left(\alpha c_D - \frac{k+2}{k+1} \eta \right) (c_D)^k + w^{-k-1} \tau^{-k} L^* \frac{Const_1}{L^* (c_M^*)^{-k} + L c_M^{-k} w^{-k} \tau^{-k}} \right] = Const_1, \text{ or}$$

$$(c_M)^{-k} \left[L \left(\alpha c_D - \frac{k+2}{k+1} \eta \right) (c_D)^k + L^* \frac{Const_1}{\tau^k w^{k+1} L^* (c_M^*)^{-k} + w L c_M^{-k}} \right] = Const_1.$$

Appendix D: Non-Wasteful Import Tariff in Small Economy

Welfare. Before deriving the optimal value of a non-wasteful import tariff, let us discuss the formula for per capita welfare at Home. As shown in Appendix A, per capita welfare can be written as a function of Q ($Q = \int_{\Omega} q^c(i) di$):

$$U = \frac{k+1}{2(k+2)} Q \left[\frac{\alpha(2k+3)}{k+1} - \eta Q \right].$$

Note that we can no longer re-write it as a function of c_D only, since as shown below, the expression for Q changes to include tariff revenues. This means that we need to study the behavior of U with respect to Q , which depends on whether

$$Q \leq \frac{\alpha(2k+3)}{\eta(2k+2)}.$$

The restriction from the model is that $\alpha - \eta Q > 0$ (so that prices in the equilibrium are non-negative). Since in any equilibrium it has to be the case that $Q < \alpha/\eta < \frac{\alpha(2k+3)}{\eta(2k+2)}$, then, as Q rises, per capita welfare at Home rises as well. Hence, in order to study the effect of a tariff on welfare at Home, it is enough to look at the behavior of Q . As discussed in the main body of the paper, welfare per capita at Home rises with Q . Note that $Q = (\alpha - w\lambda c_D)/\eta$. Hence, to see what happens with Q and welfare per capita, we need to study the behavior of $w\lambda c_D$: if $w\lambda c_D$ falls, then Q and, in turn, welfare at Home rise.

Optimal tariffs. Denote $w\lambda c_D$ by z , i.e.,

$$z \equiv w\lambda c_D.$$

Let us re-write all the equilibrium conditions as functions of w , c_D , and z . In other words, instead of keeping track of λ , we will look at z . First, we get

$$(TB) : \frac{w^{2k+1}}{t^{k+1}} (c_D)^{k+1} z = C_4, \quad (45)$$

where $C_4 = \left(\frac{c_M^* \tau^*}{c_M \tau}\right)^k \gamma B (A/B)^{k+2} / (f_e(k+1))$ is a constant. We can use this equation in the (FE) and (λ) conditions to derive

$$(FE) : \frac{C_4 t^{k+1}}{w^{2k+1}} + \frac{C_5}{w^{k+1}} = C_3, \quad (46)$$

$$(z) : z + \eta \frac{k+2}{k+1} \frac{1}{c_D} \left(\frac{(t-1)\psi}{w^{k+1}} + 1 \right) = \alpha, \quad (47)$$

where the constants are $C_3 = 2\gamma c_M^k f_e(k+1)(k+2)/L$, $C_5 = \gamma B (A/B)^{k+2} \tau^{-k}/L$, and $\psi = C_4 (c_M^* \tau^*)^{-k} / (2\gamma(k+2))$.

Note that the (FE) condition becomes very convenient, since it includes only one unknown w and the parameter t . We can apply the implicit function theorem to it to get:

$$\frac{w'}{w} = \frac{1}{t} \frac{k+1}{2k+1} \frac{1}{1 + \left(\frac{k+1}{2k+1}\right) \frac{C_5 w^k}{C_4 t^{k+1}}}.$$

Next, from the (TB) condition, we get

$$\frac{c'_D}{c_D} = \frac{1}{t} - \frac{z'}{(k+1)z} - \frac{2k+1}{k+1} \frac{w'}{w}.$$

Finally, from applying the implicit function theorem to the (z) condition, we get

$$z' - \eta \frac{k+2}{k+1} \frac{1}{c_D} \left[\frac{c'_D}{c_D} \left(\frac{(t-1)\psi}{w^{k+1}} + 1 \right) - (k+1) \frac{(t-1)\psi}{w^{k+1}} \frac{w'}{w} + \frac{\psi}{w^{k+1}} \right] = 0.$$

Using the expression for c'_D/c_D , we can re-write it as

$$z' = \frac{\frac{\psi}{w^{k+1}} \left(\frac{w'}{w} (t-1) \frac{k^2}{k+1} - \frac{1}{t} \right) + \frac{1}{t} \left(1 - t \frac{2k+1}{k+1} \frac{w'}{w} \right)}{\left(\eta_{k+1}^{k+2} \frac{1}{c_D} \right)^{-1} \left(1 + \frac{1}{k+1} \frac{1}{z} \eta_{k+1}^{k+2} \frac{1}{c_D} \left(\frac{(t-1)\psi}{w^{k+1}} + 1 \right) \right)}.$$

The denominator of z' is positive. Hence, the sign of z' depends on the sign of its nominator. Given the properties of w'/w , the second term in the nominator is always positive. However, its first term can be negative. In fact,

$$\frac{\psi}{w^{k+1}} \left(\frac{w'}{w} (t-1) \frac{k^2}{k+1} - \frac{1}{t} \right) \Big|_{t=1} = -\frac{\psi}{w^{k+1}} < 0.$$

So what can we say about the sign of z' ?

Lemma 2 *In the case of a small economy,*

$$\frac{dz}{dt} = \begin{cases} < 0, & \text{for } 1 \leq t < 1 + \frac{1}{k}, \\ = 0, & \text{for } t = 1 + \frac{1}{k}, \\ > 0, & \text{for } t > 1 + \frac{1}{k}. \end{cases}$$

Proof. First, let us introduce the following notation:

$$\tilde{r}_D = G(c_D) \bar{r}_D = \int_0^{c_D} r(c) dG(c), \quad \tilde{r}_X = G(c_X) \bar{r}_X = \int_0^{c_X} r_X(c) dG(c), \quad (48)$$

i.e., $\tilde{r}_D$ and $\tilde{r}_X$ are the expected revenues of Home firms in local and Foreign markets, respectively.

Then the (FE) condition can be re-written as

$$(FE) : \tilde{r}_D + \tilde{r}_X = w f_e(k+1). \quad (49)$$

Next, from the (FE) condition given by (46),

$$\frac{\tilde{r}_X}{\tilde{r}_D} = \frac{C_5 w^k}{C_4 t^{k+1}}, \quad \text{so that} \quad (50)$$

$$\frac{w'}{w} = \frac{1}{t} \frac{\tilde{r}_D}{\frac{2k+1}{k+1} \tilde{r}_D + \tilde{r}_X}, \quad (51)$$

and the second term in the nominator of z' is

$$\frac{1}{t} \left(1 - t \frac{2k+1}{k+1} \frac{w'}{w} \right) = \frac{1}{t} \frac{\tilde{r}_X}{\frac{2k+1}{k+1} \tilde{r}_D + \tilde{r}_X}.$$

Moreover, from (49) and (50), it can be shown directly that

$$\frac{\psi}{w^{k+1}} = \frac{\tilde{r}_X}{\tilde{r}_D + \tilde{r}_X}.$$

Hence, the nominator of z' becomes

$$\begin{aligned} & \frac{1}{t} \frac{\tilde{r}_X}{\tilde{r}_D + \tilde{r}_X} \left[\frac{(t-1)k^2}{k+1} \frac{\tilde{r}_D}{\frac{2k+1}{k+1}\tilde{r}_D + \tilde{r}_X} - 1 + \frac{\tilde{r}_D + \tilde{r}_X}{\frac{2k+1}{k+1}\tilde{r}_D + \tilde{r}_X} \right] \\ &= \frac{1}{t} \frac{\tilde{r}_X}{(\tilde{r}_D + \tilde{r}_X) \left(\frac{2k+1}{k+1}\tilde{r}_D + \tilde{r}_X \right)} \frac{k\tilde{r}_D}{k+1} [(t-1)k-1]. \end{aligned}$$

This means that $\text{sign}(z') = \text{sign}((t-1)k-1)$. Therefore, Lemma is proved. QED.

Recall that as z falls, welfare rises. Hence, we proved Proposition 2.

Appendix E: Non-Wasteful Import Tariff in Large Economy

As discussed in Appendix D, we need to study the behavior of $z \equiv w\lambda c_D$. Using the new notation and replacing $\lambda^* (c_D^*)^{k+2}$ from (32) in the (FE) , $(FE)^*$, and (λ) conditions, we get:

$$(FE): \quad z (c_D)^{k+1} \left[L + L^* \left(\frac{c_M}{c_M^* \tau^*} \right)^k \frac{w^k}{t^{k+1}} \right] = \text{Const}_1 (c_M)^k, \quad (52)$$

$$(FE)^*: \quad z (c_D)^{k+1} \left[L^* \left(\frac{c_M \tau}{c_M^* \tau^*} \right)^k \frac{w^{2k+1}}{t^{k+1}} + L (\tau^*)^{-k} \frac{w^{k+1}}{t^{k+1}} \right] = \text{Const}_1 (c_M^*)^k, \quad (53)$$

$$(z): \quad z + \frac{\eta}{c_D} \frac{k+2}{k+1} [1 + zy c_D] = \alpha, \quad (54)$$

where $y = \frac{t-1}{2\gamma(k+2)t^{k+1}} \left(\frac{w c_D}{\tau^* c_M} \right)^k \frac{L^*}{f_e(k+1)}$. Note that combining (52) and (53) gives

$$L t^{k+1} + L^* \left(\frac{c_M}{c_M^* \tau^*} \right)^k w^k = L^* \left(\frac{c_M \tau}{c_M^* \tau^*} \frac{c_M}{c_M^*} \right)^k w^{2k+1} + L \left(\frac{c_M}{c_M^* \tau^*} \right)^k w^{k+1}, \quad (55)$$

which has only one unknown w and parameter t . From the implicit function theorem, we get:

$$t \frac{w'}{w} = \left[1 + \frac{k L^* \left(\frac{c_M}{c_M^* \tau^*} \frac{c_M}{c_M^*} \right)^k w^{2k+1} + L^* \left(\frac{c_M}{c_M^* \tau^*} \right)^k w^k}{(k+1) L t^{k+1}} \right]^{-1}. \quad (56)$$

Also, by applying the implicit function theorem to the $(FE)^*$ condition, we get

$$\frac{c'_D}{c_D} = \frac{1}{t} - \frac{z'}{(k+1)z} - \frac{w'}{(k+1)w} \frac{(2k+1) L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + (k+1) L w^{k+1}}{L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + L w^{k+1}}. \quad (57)$$

Finally, from the $(FE)^*$ condition, we have

$$zy c_D = \frac{(t-1) L^*}{w \left(L + L^* \left(\frac{c_M}{c_M^*} \tau \right)^k w^k \right)}, \quad \text{so that}$$

$$(z) : z + \frac{\eta}{c_D} \frac{k+2}{k+1} \left[1 + \frac{(t-1)L^*}{w \left(L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k \right)} \right] = \alpha.$$

After applying the implicit function theorem to the equation above, we get

$$\begin{aligned} z' - \frac{\eta}{c_D} \frac{k+2}{k+1} \left[\frac{c_D'}{c_D} \left(1 + \frac{(t-1)L^*}{w \left(L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k \right)} \right) - \frac{L^*}{w \left(L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k \right)} \right. \\ \left. + \frac{(t-1)L^*}{w \left(L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k \right)^2} \frac{w'}{w} \left(L + (k+1)L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^k \right) \right] = 0. \end{aligned}$$

Using the expression for c_D'/c_D , we can re-write it as

$$\begin{aligned} z' = & \frac{\eta}{tc_D} \frac{k+2}{k+1} \left(1 + \frac{1}{(k+1)z} \eta \frac{k+2}{k+1} \frac{1}{c_D} \left(1 + \frac{(t-1)L^*}{w \left(L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k \right)} \right) \right)^{-1} \\ & * \left[\left(1 + \frac{(t-1)L^*}{w \left(L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k \right)} \right) \left(1 - \frac{tw'}{(k+1)w} \frac{(2k+1)L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + (k+1)Lw^{k+1}}{L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + Lw^{k+1}} \right) \right. \\ & \left. - \frac{tL^*}{w \left(L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k \right)} + \frac{(t-1)L^*}{w \left(L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k \right)} \frac{tw' L + (k+1)L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^k}{L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k} \right]. \end{aligned}$$

The first multipliers are positive so that to know the sign of z' , we need to look at the last term in squared brackets that can be re-written as

$$\begin{aligned} \text{sign}(z') = & \text{sign} \left[\frac{L^*}{w \left(L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k \right)} \left(\frac{(t-1)tw'}{(k+1)w} k^2 - 1 - \frac{(t-1)tw'}{(k+1)w} k^2 \frac{L}{L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k} \right) \right. \\ & \left. + 1 - \frac{tw'}{(k+1)w} \frac{(2k+1)L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + (k+1)Lw^{k+1}}{L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + Lw^{k+1}} \right]. \end{aligned}$$

Note that the (FE) condition can be written as $\tilde{r}_D + \tilde{r}_X = wf_e(k+1)$, so that from (52),

$$\frac{\tilde{r}_X}{\tilde{r}_D} = \frac{L^* w^k \left(\frac{c_M}{\tau^* c_M^*} \right)^k}{L t^{k+1}}, \quad \text{and from (56)} \quad (58)$$

$$\left[\frac{tw'}{(k+1)w} \right]^{-1} = (k+1) + \left(1 + k \left(\frac{\tau c_M}{c_M^*} \right)^k w^{k+1} \right) \frac{\tilde{r}_X}{\tilde{r}_D}.$$

Moreover, from (55),

$$\frac{L^*}{wL + L^* \left(\frac{c_M}{c_M^*} \tau \right)^k w^{k+1}} = \frac{L^* \left(\frac{c_M}{\tau^* c_M^*} \right)^k w^k}{Lt^{k+1} + L^* \left(\frac{c_M}{c_M^* \tau^*} \right)^k w^k} = \frac{\tilde{r}_X}{\tilde{r}_D + \tilde{r}_X}.$$

Finally,

$$\frac{(2k+1) L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + (k+1) L w^{k+1}}{L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + L w^{k+1}} = (k+1) + k w^{k+1} \left(\frac{\tau c_M}{c_M^*} \right)^k \frac{\tilde{r}_X}{\tilde{r}_D + \tilde{r}_X}.$$

Using all the relationships together, we get

$$\begin{aligned} \text{sign}(z') &= \text{sign} \left[\frac{\tilde{r}_X}{\tilde{r}_D + \tilde{r}_X} \frac{tw'}{(k+1)w} (t-1) k^2 \left(1 - \frac{L}{L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k} \right) + \frac{\tilde{r}_D}{\tilde{r}_D + \tilde{r}_X} \right. \\ &\quad \left. - \frac{tw'}{(k+1)w} \left((k+1) + k w^{k+1} \left(\frac{\tau c_M}{c_M^*} \right)^k \frac{\tilde{r}_X}{\tilde{r}_D + \tilde{r}_X} \right) \right] \\ &= \text{sign} \left[\frac{\tilde{r}_X}{\tilde{r}_D + \tilde{r}_X} \frac{tw'}{(k+1)w} \left((t-1) k^2 \frac{L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k}{L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k} - (k+1) \frac{\tilde{r}_D}{\tilde{r}_X} + 1 + k \left(\frac{\tau c_M}{c_M^*} \right)^k w^{k+1} \right. \right. \\ &\quad \left. \left. - (k+1) \left(\frac{\tilde{r}_D}{\tilde{r}_X} + 1 \right) - k w^{k+1} \left(\frac{\tau c_M}{c_M^*} \right)^k \right) \right]. \end{aligned}$$

By noting that $w' > 0$ and rearranging the term in the brackets, we have

$$\text{sign}(z') = \text{sign} \left[\left((t-1) k^2 \frac{L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k}{L + L^* \left(\frac{c_M \tau}{c_M^*} \right)^k w^k} - k \right) \right],$$

which means that

$$\frac{dz}{dt} = \begin{cases} < 0, & \text{for } 1 \leq t < t^{opt}, \\ = 0, & \text{for } t = t^{opt}, \\ > 0, & \text{for } t > t^{opt}, \end{cases}$$

where

$$t^{opt} = 1 + \frac{1}{k} + \frac{1}{k} \frac{L}{L^*} \left(\frac{c_M^*}{c_M \tau} \right)^k w^{-k}.$$

Note that the equation above still needs to be solved, since w is a function of t that is implicitly defined by (55). Given (56), w rises with t , meaning that the solution of the equation above is unique. Finally, recall that as z falls, welfare rises. Hence, we proved Proposition 3.

No Metzler paradox. We want to study the effect of a falling non-wasteful import tariff t on the average prices at Home and abroad in the case of two large economies. Given the Pareto cost distribution assumption, these prices can be written as, respectively,

$$\bar{p} = \frac{2k+1}{2(k+1)}wc_D \quad \text{and} \quad \bar{p}^* = \frac{2k+1}{2(k+1)}c_D^*.$$

First, let us look at wc_D . From (57) and (56),

$$\begin{aligned} (wc_D)' &= wc_D \left[\frac{c_D'}{c_D} + \frac{w'}{w} \right] \\ &= wc_D \left[\frac{1}{t} - \frac{z'}{(k+1)z} + \frac{w'}{w} \left(1 - \frac{(2k+1)L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + (k+1)Lw^{k+1}}{(k+1) \left(L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + Lw^{k+1} \right)} \right) \right] \\ &= wc_D \left[-\frac{z'}{(k+1)z} + \frac{1}{t} \left(1 - \frac{kL^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1}}{(k+1) \left(L^* \left(\frac{\tau c_M}{c_M^*} \right)^k w^{2k+1} + Lw^{k+1} \right)} t \frac{w'}{w} \right) \right], \end{aligned}$$

where for $t \in [1, t^{opt}]$, $z'/z < 0$, $0 < tw'/w < 1$, and the multiplier in front of tw'/w is positive and less than 1. Hence, we get

$$\frac{d\bar{p}}{dt} = \frac{2k+1}{2(k+1)} \frac{d(wc_D)}{dt} > 0,$$

i.e., the average price at Home rises with t . Next, let us look at c_D^* . From (32),

$$\lambda^* (c_D^*)^{k+2} = (c_D^*)^k \left(\alpha c_D^* - \frac{k+2}{k+1} \eta \right) = \frac{w^{2k+1}}{t^{k+1}} z (c_D)^{k+1} \left(\frac{\tau c_M}{\tau^* c_M^*} \right)^k,$$

where the left-hand side is monotonically increasing in c_D^* . From (53),

$$\frac{w^{2k+1}}{t^{k+1}} z (c_D)^{k+1} \left(\frac{\tau c_M}{\tau^* c_M^*} \right)^k \left[L^* + L^* \left(\frac{c_M \tau}{c_M^* \tau^*} \right)^{-k} (\tau^*)^{-k} \frac{1}{w^k} \right] = Const_1 (c_M^*)^k.$$

Given that w rises with t , the term in the squared brackets above falls, so that the multiplier in front of these brackets has to rise, meaning that c_D^* rises too. Thus, we have

$$\frac{d\bar{p}^*}{dt} = \frac{2k+1}{2(k+1)} \frac{d(c_D^*)}{dt} > 0,$$

so that the average prices everywhere rise with an increase in t , implying no Metzler paradox in our model.

Appendix F: Role of Variable Markups

A rise in misallocation distortion in the case of falling trade costs. Here we show that covariance in (39) is negative for a marginal fall in τ^* .

$$\begin{aligned} \text{cov} \left(m(\omega, i), \frac{dl(\omega, i)}{L_j} \right) &= M \int_0^{c_D} \frac{p(c)}{wc} \frac{d[cq(c)]}{L} \frac{dG(c)}{G(c_D)} + M_x \int_0^{c_x} \frac{p_X(c)}{w\tau c} \frac{d[\tau cq_X(c)]}{L} \frac{dG(c)}{G(c_X)} \\ &= \frac{M}{wL} \int_0^{c_D} p(c) d[q(c)] \frac{dG(c)}{G(c_D)} + \frac{M_x}{wL} \int_0^{c_x} p_X(c) d[q_X(c)] \frac{dG(c)}{G(c_X)} \end{aligned} \quad (59)$$

First, note that

$$dq(c) = d \left(\frac{L}{2\gamma} w\lambda (c_D - c) \right) = \frac{L}{2\gamma} [d(w\lambda c_D) - cd(w\lambda)],$$

so that using the direct calculations for the Pareto cost distribution, we get

$$\frac{M}{wL} \int_0^{c_D} p(c) d[q(c)] \frac{dG(c)}{G(c_D)} = \frac{1}{4\gamma wL} \frac{ML}{(k+1)(k+2)} \left[(2k+1)(k+2) c_D d(w\lambda c_D) - (2k^2 + 3k) (c_D)^2 d(w\lambda) \right].$$

Similarly (here we use $d\tau = 0$),

$$\begin{aligned} \frac{M_x}{wL} \int_0^{c_x} p_X(c) d[q_X(c)] \frac{dG(c)}{G(c_X)} &= \frac{1}{4\gamma wL} \frac{M_X L^*}{(k+1)(k+2)} \\ &\quad * \left[(2k+1)(k+2) \tau^2 c_X d(w\lambda^* c_X) - (2k^2 + 3k) (\tau c_X)^2 d(w\lambda^*) \right]. \end{aligned}$$

Hence, the expression for the covariance of markups and labor share changes is

$$\begin{aligned} \text{cov} \left(m(\omega, i), \frac{dl(\omega, i)}{L_j} \right) &= \frac{(2k+1)}{4\gamma wL (k+1)} [ML c_D d(w\lambda c_D) + M_X L^* \tau^2 c_X d(w\lambda^* c_X)] \\ &\quad - \frac{(2k^2 + 3k)}{4\gamma wL (k+1)(k+2)} [ML (c_D)^2 d(w\lambda) + M_X L^* (\tau c_X)^2 d(w\lambda^*)]. \end{aligned}$$

As noted in Section 5, the total labor size at Home does not change, which implies $\sum_i \int_{\omega \in \Omega_{ji}} \left[\frac{dl(\omega, i)}{L_j} \right] d\omega = 0$. We can re-write it as

$$\begin{aligned} M \int_0^{c_D} d[cq(c)] \frac{dG(c)}{G(c_D)} + M_x \int_0^{c_x} d[\tau cq_X(c)] \frac{dG(c)}{G(c_X)} &= 0, \quad \text{or} \\ \frac{1}{2\gamma (k+1)(k+2)} ML [k(k+2) c_D d(w\lambda c_D) - k(k+1) c_D d(w\lambda)] \\ + \frac{1}{2\gamma (k+1)(k+2)} M_X L^* [k(k+2) \tau^2 c_X d(w\lambda^* c_X) - k(k+1) (\tau c_X)^2 d(w\lambda^*)] &= 0. \end{aligned}$$

Multiplying the expression above by $2\gamma(2k+3)$ and rearranging the terms result in

$$\frac{(2k^2 + 3k)}{(k+2)} [ML c_D d(w\lambda) + M_X L^* (\tau c_X)^2 d(w\lambda^*)] = \frac{k(2k+3)}{k+1} [ML c_D d(w\lambda c_D) + M_X L^* \tau^2 c_X d(w\lambda^* c_X)].$$

We can use this equation in the expression for the covariance to get:

$$\begin{aligned} cov\left(m(\omega, i), \frac{dl(\omega, i)}{L_j}\right) &= \frac{1}{4\gamma wL(k+1)} \left[(2k+1) - \frac{k(2k+3)}{k+1} \right] [MLc_D d(w\lambda c_D) + M_X L^* \tau^2 c_X d(w\lambda^* c_X)] \\ &= \frac{1}{4\gamma wL(k+1)^2} [MLc_D d(w\lambda c_D) + M_X L^* \tau^2 c_X d(w\lambda^* c_X)]. \end{aligned}$$

Finally, note that from (20) and (12), we get

$$\begin{aligned} d(w\lambda c_D) &= d\left(\alpha - \frac{k+2}{k+1} \eta \frac{1}{c_D}\right) = \frac{k+2}{k+1} \eta \frac{dc_D}{(c_D)^2}, \quad \text{and} \\ d(w\lambda^* c_X) &= d\left(\frac{1}{\tau} \lambda^* c_D^*\right) = \frac{1}{\tau} \frac{k+2}{k+1} \eta \frac{dc_D^*}{(c_D^*)^2}. \end{aligned}$$

Then

$$cov\left(m(\omega, i), \frac{dl(\omega, i)}{L_j}\right) = \frac{(k+2)\eta}{4\gamma wL(k+1)^3} \left[ML \frac{dc_D}{c_D} + M_X L^* \frac{dc_D^*}{wc_D^*} \right].$$

From Proposition 1, when τ^* falls, c_D and c_D^* fall as well. Hence, the expression above is negative for $d\tau^* < 0$. QED.

A fall in the average markup in the case of a rising import tariff. The average markup at Home can be written as

$$\begin{aligned} \bar{m} &= (M + M_x^*)^{-1} M \int_0^{c_D} m(c) \frac{dG(c)}{G(c_D)} + M_X^* \int_0^{c_x^*} m_X^*(c) \frac{dG^*(c)}{G^*(c_x^*)} \\ &= (M + M_x^*)^{-1} M \int_0^{c_D} \frac{1}{2} \left(\frac{c_D + c}{c} \right) \frac{kc^{k-1}dc}{(c_D)^k} + M_X^* \int_0^{c_x^*} \frac{1}{2} t \left(\frac{c_x^* + c}{c} \right) \frac{kc^{k-1}dc}{(c_x^*)^k} \\ &= \frac{1}{2} \frac{2k-1}{k-1} \frac{M + tM_x^*}{M + M_x^*} = \frac{1}{2} \frac{2k-1}{k-1} \frac{1 + t^{1-k}w^k}{1 + t^{-k}w^k} = \frac{1}{2} \frac{2k-1}{k-1} \frac{t^{k+1} + w^k}{t^{k+1} + tw^k}. \end{aligned}$$

Then, it can be shown straightforwardly that

$$\frac{d\bar{m}}{dt} = \frac{2k-1}{2(k-1)(t^{k+1} + tw^k)^2} \left[(1-t)kw^{k-1}t^{k+1} \frac{dw}{dt} - w^{2k} - kt^k w^k \left(1 + \frac{1}{k} - t \right) \right].$$

When t starts to rise from 1 (consider t between 1 and $1 + \frac{1}{k}$), w rises too (see (56)), so that for small increases in t we get

$$\frac{d\bar{m}}{dt} < 0.$$