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Learning Efficiency Shocks, Knowledge Capital and the Business Cycle : A Bayesian Evaluation

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Abstract

We incorporate shocks to the efficiency by which firms learn from production activity and accumulate knowledge into an otherwise standard real DSGE model with imperfect competition. Using real aggregate data from the United States and Bayesian inference techniques, we find that learning efficiency shocks are an important source of observed variation in the growth rate of aggregate consumption and hours worked in post-war US data. The estimated shock processes suggest much less exogenous variation in preferences and total factor productivity are needed by our model to account for the joint dynamics of consumption and hours. This occurs because learning efficiency shocks induce shifts in labour demand uncorrelated with current TFP, a role usually played by preference shocks. At the same time, knowledge capital acts like an endogenous source of productivity variation in the model. Measures of model fit prefer the specification with learning efficiency shocks.

Keywords: Business Cycles; Learning-by-Doing; Learning Efficiency Shocks; Knowledge Capital.

JEL Classification: E32.

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1 Introduction

It is well known that there are vast differences in the productivity of organizations within industries and even among production units within the same organization. These differences come about from the way in which observable inputs are combined together with existing technology to produce output. Prescott and Visscher (1980) think of this as an accumulated, unmeasured form of capital which they call “organizational capital”.¹ Others have referred to it as “knowledge capital”, a term that we will use in this paper.² Organizational or knowledge capital is an umbrella term that captures many diverse phenomena including a work culture that induces extra unobserved effort or motivation from employees or a culture that recognizes mistakes and quickly moves to remedy them. It also includes firm level knowledge regarding how best to deploy workers to tasks, or how best to use teams within the organization - details which are lost in the homogeneous worker environment typically found in aggregate macro models designed to explain aggregate data.

Work on organizational capital is related to measurement of the sources of productivity increases within organizations. For example, since the work of Arrow (1962) and Rosen (1972), economists have also collected evidence that productivity increases with experience, a process that is called “learning-by-doing” (see Bahk and Gort (1993) and Benkard (2000) for some evidence and references to a vast literature). Rather than focusing on any particular way in which firms create productivity enhancing knowledge in-house (by experimenting with the production process or through learning as a by-product of experience with production itself), several papers have studied the impact of the presence of these types of knowledge capital in explaining a number of business cycle phenomena in a dynamic general equilibrium

¹Atkeson and Kehoe (2005) develop a model that allows measurement of organizational capital at the aggregate level.

²A number of similar labels are used for this type of firm level knowledge that influences productivity: knowledge capital, intangible capital, managerial capital, organizational capital. One common distinction between these labels is whether resources have to be explicitly set aside to create the knowledge or if it can be accumulated as a by-product of productive activity. We focus on the latter class of models here.

context. For example, knowledge capital has been used to explain the propagation of business cycle shocks (Cooper and Johri, 2002), endogenous variations in total factor productivity (TFP) (Clarke and Johri, 2009), as a source of sticky prices and inflation persistence in a monetary model (Johri, 2009), as a source of co-movement domestically in response to news shocks (Gunn and Johri, 2011) and across borders in a two-country RBC model (Johri et al., 2011) and as a source of real exchange rate persistence in Johri and Lahiri (2008).

A common feature of these models with knowledge capital is that agent's learning-functions are non-stochastic. The amount of new knowledge created is proportional to either the amount of labour in use or the amount of production activity. If however, knowledge capital is created by the process of experimentation with existing inputs, one might expect some variation in the results - some experiments lead to large increases in productivity and are incorporated into the internal body of knowledge while others are unsuccessful and abandoned. Even with the more traditional, deterministic learning-by-doing environment, we might argue that the learning function is concave for individual agents, so that the aggregate learning function may display variations in the amount learned from the same amount of activity in different time periods depending on the micro-level distribution of agents in terms of their location on the learning curve.³ For example, when more agents are at an early stage of their economic projects, aggregate learning rates may be higher than times with few agents in early stages. Rather than focusing on the sources of this randomness, the goal of this paper is to incorporate randomness in the knowledge capital accumulation process and assess whether this new source of shocks helps the business cycle model to account for variations in real aggregate data. We will refer to these new disturbances to the model as learning efficiency shocks (LES).

Similar to the literature discussed above, we incorporate knowledge capital into the pro-

³For evidence on the empirical concavity of learning functions see: Jovanovic and Nyarko (1995) and Irwin and Klenow (1994).

duction technology of intermediate goods firms in a standard dynamic stochastic general equilibrium (DSGE) model with imperfect competition. Unlike those papers, in our model, these firms face shocks to the efficiency with which they accumulate knowledge capital. Using Bayesian inference methods and US data on the real growth rate of output, consumption and hours worked, we estimate the key parameters of the model as well as various shocks hitting the economy. Besides learning efficiency shocks, we use two other structural shocks that are often used in the business cycle literature: stationary TFP shocks and preference shocks.⁴

We find that learning efficiency shocks are an important source of observed fluctuations in the real variables of interest. Our variance decomposition results show that they explain over 40 percent of the observed variance in consumption growth and 20-60 percent of the variance in hours even in the presence of preference shocks. Compared to an estimated standard DSGE model with imperfect competition without knowledge capital and even to a model with knowledge capital but without learning efficiency shocks, our proposed model (with both knowledge capital and learning efficiency shocks) delivers better results as captured by the log-data-density values (marginal likelihood). Impulse responses of our model to learning efficiency shocks help to uncover the mechanism by which these shocks work and why they explain movements in hours and consumption. Shocks to learning efficiency create shifts in labour demand which are not driven by contemporaneous changes in TFP. As is well known, business cycle models typically have a hard time explaining the joint dynamics of consumption, productivity and hours. A common solution to this problem has been the introduction of exogenous shifts in preferences. The endogenous movements in labour demand introduced by learning efficiency shocks create a wedge between current productivity and hours worked, thus necessitating less exogenous shifts in labour supply through prefer-

⁴We work with a small scale model with two or three real variables in order to isolate the impact of the novel learning efficiency shocks without confounding interactions with other frictions and shocks found in the typical new keynesian model as in Christiano et al. (2005) and Smets and Wouters (2007).

ence shocks in order to capture the variation seen in the data. We find that the estimated variance (at the mode) of preference shocks falls by 33.33 % as we go from the no knowledge capital model to the model with learning efficiency shocks. The estimates also imply that the TFP process is much less persistent than suggested by the model without knowledge capital. This in turn implies less volatility in the level of estimated TFP which falls by 99.69 % as we move from the model without knowledge capital to the model with learning efficiency shocks. The remaining variance of TFP is still able to account for about 7% to 15% of the variation (depending on the horizon) in the growth rate of hours worked due to the internal propagation built in the model.

The rest of the paper is structured as follows. Section 2, presents the model and defines the equilibrium. In Section 3, we discuss our empirical results. Section 4 concludes.

2 Model

The model economy consists of a representative household and a representative final good producer for notational simplicity. In addition, there is a continuum of intermediate goods producers operating in a Dixit-Stiglitz style market with imperfect competition.

2.1 Household's Problem

The representative household is endowed with a unit of time each period which is allocated to leisure and work. Substituting out leisure from the problem, the household maximizes its expected discounted life-time utility from consumption, C_t , and labour supply, N_t :

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

where E_0 is the conditional expectations operator, $\beta \in (0, 1)$ is the household's subjective

discount factor, and $U(C_t, N_t)$ has a log-linear functional form:

$$U(C_t, N_t) = \theta_t \log(C_t) + \psi(1 - N_t) \quad (2)$$

Here θ_t is a preference shock and ψ is a parameter governing the weight of leisure in the period utility. The preference shock expressed in log deviations from steady state follows a standard AR(1) process with *iid* innovations:

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} + \epsilon_{\theta,t} \quad \epsilon_{\theta,t} \sim iid(0, \sigma_\theta). \quad (3)$$

In each period, the representative household supplies labour at a wage rate, w_t , and it receives an equity payout Π_t for each unit of its outstanding holdings of intermediate firm equity S_t . Anticipating a symmetric equilibrium in which all firms behave the same, we suppress notation for individual firms' equity and dividend payouts. The household allocates its current income between consumption and total equity. Its sequence of budget constraints is given by

$$C_t + p_t S_{t+1} = w_t N_t + S_t(p_t + \Pi_t) \quad (4)$$

where p_t is the price of equity.

The household's problem is to choose contingent sequences, $\{C_t, N_t, S_{t+1}\}$ to maximize the objective function (1) subject to the budget constraint (4). The first order conditions of this problem are:

$$U_{c,t} = \lambda_t \quad (5)$$

$$-U_{n,t} = \lambda_t w_t \quad (6)$$

$$p_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} [\Pi_{t+1} + p_{t+1}] \right\} = \sum_{j=1}^{\infty} E_t \left\{ \beta^j \frac{\lambda_{t+j}}{\lambda_t} \Pi_{t+j} \right\} \quad (7)$$

where λ_t is the Lagrange multiplier on constraint (4), $U_{n,t}$ is the marginal disutility from labour, and $U_{c,t}$ is the marginal utility of consumption.

2.2 Final Good Producer

In order to economize notation we will assume that there is a single representative final good producer who nonetheless behaves competitively. The final good technology uses $y_t(i)$ units of a continuum of intermediate good $i \in [0, 1]$, to produce Y_t units of the final good in period t . If we assume that all intermediate goods are imperfect substitutes with a constant elasticity of substitution, $\frac{1}{1-\eta}$, then the corresponding Dixit-Stiglitz aggregator is

$$Y_t = \left[\int y_t(i)^\eta di \right]^{\frac{1}{\eta}}, \quad 0 < \eta < 1 \quad (8)$$

The objective of the final-good producer is to maximize its profits by choosing the quantity of each intermediate good $y_t(i)$.

$$\max_{y_t(i)} Y_t - \int q_t(i) y_t(i) di \quad (9)$$

subject to (8)

where $q_t(i)$ is the relative price charged by the i^{th} intermediate good producer. Profit maximization implies the following input demand functions:

$$y_t(i) = q_t(i)^{-\frac{1}{1-\eta}} Y_t \quad \forall i \in [0, 1] \quad (10)$$

where $\frac{1}{1-\eta}$ represents the price elasticity of demand for intermediate good i .

2.3 Intermediate Good Producers

There is a continuum of intermediate good producers indexed by $i \in [0, 1]$. Firm i produces the differentiated good i using a constant returns to scale technology which uses labour $N_t(i)$, physical capital, $K_t(i)$ and knowledge capital $Z_t(i)$ as inputs:

$$y_t(i) = (A_t N_t(i))^\alpha (K_t(i))^{1-\alpha-\varepsilon} Z_t(i)^\varepsilon. \quad (11)$$

A_t represents a stationary technology shock.⁵ Expressed in log deviations from steady state, it follows a first-order autoregressive process with an *iid* innovation:

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{A,t} \quad \epsilon_{A,t} \sim iid(0, \sigma_A) \quad (12)$$

The intermediate good producer accumulates both physical capital and knowledge capital. Its physical capital evolves according to

$$K_{t+1}(i) = I_t(i) + (1 - \delta)K_t(i) \quad (13)$$

where δ is the depreciation rate of physical capital and I_t refers to investment. Knowledge capital accumulates according to

$$Z_{t+1}(i) = B_t(N_t(i))^{(1-\gamma)} Z_t(i)^\gamma \quad (14)$$

Equation (14) is interpreted as the firm combining its existing knowledge and hours of work to produce more knowledge. The parameter $\gamma \in (0, 1)$ governs the relative contribution of past knowledge to current learning.

⁵The model implies that the long-term growth rate of output and consumption are equal while they are slightly different in the data. In order to prevent these counter-factual model implications from biasing our estimation results and the higher frequency business cycle properties of the model, we abstract from all sources of long run growth here.

As discussed in the introduction, the shock B_t in equation (14) captures the inherent randomness associated with a firm's ability to create knowledge capital. It follows a first-order autoregressive process with an *iid* innovation

$$\ln B_t = \rho_B \ln B_{t-1} + \epsilon_{B,t} \quad \epsilon_{B,t} \sim iid(0, \sigma_B) \quad (15)$$

Each firm issues one unit of equity on which it pays dividends every period. Defining dividends of the firm i as $\Pi_t(i) = q_t(i)y_t(i) - w_t N_t(i) - I_t(i)$, in each period, it must choose contingency plans for $\{q_t(i), y_t(i), N_t(i), I_t(i), K_{t+1}(i), Z_{t+1}(i)\}_{t=0}^{\infty}$ to maximize the present value of real dividends:

$$\mathcal{V}_0(i) = \max E_0 \sum_{t=0}^{\infty} D_t \left\{ \Pi_t(i) \right\} \quad (16)$$

subject to (10), (11), (13), (14) and appropriate transversality conditions on the stock of physical capital and knowledge capital. The variable $D_t = \beta \frac{U_{c,t}}{U_{c,0}}$ is the endogenous discount factor for firm i . The first order conditions are given by:

$$y_t(i) = \lambda_t^q(i) \left(\frac{1}{1-\eta} \right) \frac{y_t(i)}{q_t(i)} \quad (17)$$

$$\lambda_t^K(i) = E_t \left[\frac{D_{t+1}}{D_t} \left((1-\alpha-\varepsilon) \lambda_{t+1}^y(i) \frac{y_{t+1}(i)}{K_{t+1}(i)} + \lambda_{t+1}^K(i) (1-\delta) \right) \right] \quad (18)$$

$$w_t(i) = \lambda_t^y(i) \alpha \frac{y_t(i)}{N_t(i)} + \lambda_t^Z(i) (1-\gamma) \frac{Z_{t+1}(i)}{N_t(i)} \quad (19)$$

$$\lambda_t^Z(i) = E_t \left[\frac{D_{t+1}}{D_t} \left(\varepsilon \lambda_{t+1}^y(i) \frac{y_{t+1}(i)}{Z_{t+1}(i)} + \lambda_{t+1}^Z(i) \gamma \frac{Z_{t+2}(i)}{Z_{t+1}(i)} \right) \right] \quad (20)$$

where λ_t^q , λ_t^y , λ_t^K and λ_t^Z are the Lagrangian multipliers associated with constraints (10), (11), (13), (14), respectively. Equation (17) implies that an intermediate-good producer chooses its relative price for its differentiated goods as a constant mark-up over the real

marginal cost. Equation (18) reflects the standard calculus facing firms that accumulate physical capital.

The impact of the presence of knowledge capital in the firm's problem is seen in equation (19) where the second term on the right side of the equation reflects the contribution of hiring an additional unit of labour to future knowledge capital. It implies that the price of labour is higher than the value of the extra output currently produced by the firm as would be the case in a standard model with imperfect competition. This "extra" labour hired by the firm may be seen as a form of investment in knowledge capital since the wage bill is not covered by the current revenues generated from selling output.

Equation (20) equates the marginal value of an extra unit of knowledge capital to the benefit that comes not only from the extra production of the final good made possible but also from the additional knowledge capital that can be produced in the future since it is an input in both the creation of goods as well as new knowledge capital.

2.4 Equilibrium

Equilibrium in this model is a sequence of allocations for households: $\{C_t, N_t, S_{t+1}\}_{t=0}^{\infty}$, allocations for intermediate goods producers: $\{Z_{t+1}(i), N_t(i), K_{t+1}(i), I_t(i)\}_{t=0}^{\infty}$ for all $i \in [0, 1]$, allocations for final goods producers: $\{Y_t, y_t(i)\}_{t=0}^{\infty}$ for all $i \in [0, 1]$ together with prices $\{w_t, q_t(i)\}_{t=0}^{\infty}$ that satisfy:

- Given prices, the allocations of households solve the household problem.
- Given prices, the allocation of final goods-producers solve the final goods producer problem.
- Given all prices except for own price, the allocations of the intermediate goods producers solve the intermediate goods producer problem. Own price is chosen to maximize expected discounted dividends.

- The market for input factors and for final goods clear.
- The market-clearing condition in the share market requires that $S_t = 1$ and hence, consumption at period t , C_t , equals all the income earned in period t .
- Since all the output produced, is either invested or given as dividends to household, then the resource constraint is satisfied:

$$Y_t = C_t + I_t \quad (21)$$

- The market-clearing condition for labour requires that the total demand for labour from firms is equal to the amount supplied by the representative household:

$$N_t^D = \int_0^1 N_t(i) di = N_t \quad (22)$$

Moreover, we normalize the price of the final good to one and assume a symmetric equilibrium. That is, all intermediate good producers are identical. They face identical sequences of aggregate shocks $\{A_t, B_t\}_{t=0}^\infty$, start out with identical quantities of K_0 and Z_0 , charge the same relative price, and produce the same quantity of intermediate output.

Therefore,

$$Y_t = \left[\int y_t(i)^\eta di \right]^{\frac{1}{\eta}} = y_t$$

and the relative price $q_t = 1$ for all firms. Also, $N_t(i) = N_t$, $K_t(i) = K_t$, $I_t(i) = I_t$, $Z_t(i) = Z_t$ and $\Pi_t(i) = \Pi_t$.

2.5 The No Knowledge Capital (No-KC) Model

This model is very similar to the baseline (which has both KC and LES). It has the same household problem and final good producer's problem. However, the intermediate good producers do not have the ability to produce or accumulate knowledge capital. Thus, knowledge capital, Z_t , does not appear in the production technology of the intermediate good producers and there is no accumulation equation for it. The production technology of the intermediate good producers under this case is given by

$$y_t(i) = (A_t N_t(i))^\alpha (K_t(i))^{1-\alpha} \quad (23)$$

where variables and parameters have the same interpretation as before. Since this problem is a special case of the previous one and well understood, we skip the first order conditions and any discussion of the model.

3 Empirical Results

We use Bayesian methods to infer model parameters, estimate shocks and obtain measures of model fit with the data. We begin by describing the data used in our study, we then discuss which priors are used and which parameters are calibrated. Next we report our posterior estimates for parameters as well as discuss how well each model performs.

3.1 Data

We use quarterly observations on three key macroeconomic variables for the United States: the aggregate growth rates of real GDP, real consumption, and hours worked. The data sample is taken from the Federal Reserve Bank of Saint Louis' FRED database and it covers the period 1947QI to 2014QII. In particular, we use "Real Gross Domestic Product", "Real

Personal Consumption Expenditures” and ”Nonfarm Business Sector: Hours of All Persons” series. The first two are from the U.S. Bureau of Economic Analysis, and the later is from the U.S. Bureau of Labor Statistics. Their respective identification numbers are GDPC1, PCECC96, and HOANBS. All three series are seasonally adjusted and chained in 2009 dollars. The data are demeaned. We impose on the model-generated time series of interest the same transformations that are applied to the data.

3.2 Calibration and Priors Specification

As is common in the empirical DSGE literature, we keep a number of parameters fixed in our estimation. Smets and Wouters (2003) point out that some of the parameters are directly related to the steady state values of state variables and can be estimated from the means of observable variables. Since we are using demeaned observable variables then the estimation cannot pin down these parameters. We calibrate the discount factor β to be 0.99, so that our models imply an annual steady-state real interest rate of 4%. The depreciation rate on physical capital, δ , is set to 0.020, which implies an annual rate of depreciation of 8%. The parameter η is set to 0.95, which implies a 5% mark-up over the real marginal cost of intermediate good i producer. We calibrate the leisure preference parameter, ψ , to 2.39, so that the model yields a steady-state fraction of hours worked of 0.30. A summary of our calibration is presented in Table 1.

Table 1: Calibrated Parameters

Parameter	Values	Description
β	0.99	discount factor
δ	0.02	depreciation rate
η	0.95	markup parameter
ψ	2.39	leisure preference parameter

The first three columns of Tables 2, 3, 4 and 5 contain our assumptions regarding the prior

distribution of the remaining parameters of each of the four model variants that we estimate: the No-KC model, the KC model, and the KC model with learning efficiency shock and KC model with learning efficiency shock using three observable variables, respectively. For consistency reasons, in all four cases, we assume the same prior distribution for the estimated parameters. For the prior distribution of the labour share parameter (α) in the production technology, the knowledge capital share parameter (ϵ), and knowledge capital retention rate parameter (γ), we assume a beta distribution with mean 0.6, 0.25, and 0.8, and a common standard error of 0.1, respectively⁶. For the distribution of the persistence parameters in the TFP, preference and learning efficiency shocks, we assume a beta distribution with mean 0.85 and standard error 0.05. For all the variances of the shocks, we assume inverse gamma distribution with mean 0.01 and infinite standard error.

3.3 Posterior Estimates

Tables 2, 3, 4 and 5 also present the details of the parameter estimates such as the estimated posterior mode, approximate standard error, the 10th, 50th, 90th percentile of posterior distribution, and the posterior mean of the parameters. Our estimation is based on a sample of 300,000 draws that were sufficient to yield convergence of the Metropolis-Hastings sampling algorithm. We also tried larger samples with virtually no changes in results. Our posterior distributions are based on the growth rate of real consumption and hours worked for the first three models. Since the third model has an additional shock we also carry out a fourth exercise with the addition of the growth rate of real GDP as an additional observable variable.

Our posterior results show that knowledge capital plays an important role in the production of output. The estimated mode for the knowledge capital share, ϵ , in the model

⁶We set the prior mean of knowledge capital retention rate, γ , to 0.8 which is close to the value (0.797) estimated by Chang et al. (2002).

without learning efficiency shocks is 0.24 which is close to earlier estimates in the literature.⁷ The addition of learning efficiency shocks causes little change in this parameter which ranges between 0.19 – 0.27 depending on the observable variables used. The posteriors for γ , in the latter three cases are also in line with the literature. Given the discussion in Cooper and Johri (2002) regarding organizational capital as an endogenous source of propagation and magnification of shocks, it is not surprising to see that the persistence and volatility of the estimated level of TFP drop as we go from the no KC model through the KC with LES model (ρ_A at the posterior mode falls from 0.94 to 0.91 to 0.86). While not reported in these tables the volatility of estimated TFP drops by 99.69 % as we move from the model without knowledge capital to the model with learning efficiency shocks. Similarly, the model requires much smaller shifts in preferences in order to match the data, a fall of 33.33%. Recalling equation (19) and equation (20), the presence of knowledge capital creates additional terms in the labour demand equation relative to a model without knowledge capital which may be seen as a dynamic shift factor J_t

$$w_t(i) = \lambda_t^y(i) \alpha \frac{y_t(i)}{N_t(i)} + J(\lambda_t^Z; Z_t; N_t, B_t)$$

Any shock in the economy that affects the value of knowledge capital in period t causes firms to respond in a forward looking way when choosing the labour input as if the labour demand had shifted. In addition to this effect present in previous knowledge capital models, a shock to learning efficiency creates an additional direct desire for the firm to modify the amount of labour used and therefore the learning efficiency shock acts like a direct shift in labour demand. The impact of learning efficiency shocks can be seen in Figure 1, which

⁷Our posterior estimates of the labour share parameter α are low. To verify that these low estimates of labour share are not crucial for the results that follow, we reestimate the model holding α constant. We calibrated $\alpha = 0.45$ in order to obtain a steady state capital output ratio of 10, a value commonly used in the business cycle literature. Our estimation results for this case are available from the authors but are not reported here since the model estimates are very similar and imply similar measures of fit to the data as those reported below.

Figure 1: Dynamic Responses to Learning Efficiency Shock: All responses are displayed relative to the size of the shock.

displays a 1% increase in B_t starting from steady state. Firms realize that this is a good time to produce knowledge capital and immediately hire additional labour. This causes output to increase. The additional labour as well as the higher learning efficiency unleashed by the shock both result in additional knowledge capital in the next period and beyond which raises the marginal product of other inputs inducing additional increases in output. The household, anticipating the boom in income through a higher wage bill, immediately increases consumption which stays above steady state for more than 40 quarters. In Figure 1 we also plot the response of a naively constructed solow residual (sr) measured as $sr_t = y_t - \alpha n_t - (1 - \alpha)k_t$ where all lower case variables are in logs. A statistician, unaware of the presence of knowledge capital in firm production functions, would observe a rise in the solow residual in period 1 and infer the presence of a TFP shock where there was none.

As shown in Figures A1 and A2 in the appendix, both the stationary TFP shock and the preference shock create movement in the shadow price of knowledge capital as well as additional knowledge capital through changes in the labour input. Movements in N_t , λ_t^Z and Z_t create an endogenous movement in J_t on impact, even though neither the TFP nor the preference shocks directly affect it. This implies that the presence of knowledge capital induces endogenous shifts in labour demand which help propagate both of these shocks. Obviously the naive solow residual also responds to these movements in knowledge capital.

We also calculated standard theoretical second moments using parameter values at the mode of the posterior distributions. These moments display the usual business cycle patterns: investment is more volatile than output and consumption is less volatile while both variables display a high contemporaneous correlation with output.

3.4 Model fit

Following the literature, we report model fit using the log data density. These results are reported in Table 6. Starting with the two models without learning efficiency shocks estimated using the growth rate of consumption and hours, we find that the introduction of knowledge capital helps fit the model to the data. The log-data-density values for the two models are 1779.778 and 1788.561, respectively. The introduction of learning efficiency shocks raises this value to 1850.980. We conclude from these results that learning efficiency shocks add substantially to the econometric fit of the model.

3.5 The importance of learning efficiency shocks

In order to show that learning efficiency shocks are an important source of observed business cycle fluctuations we compute the posterior variance decomposition (the contribution of each shock to the unconditional variance of the observable variables). Tables 7, 8, and 9 present

the posterior variance decomposition (at various horizons) for the No-KC, KC, and KC with learning efficiency shock (LES) models, respectively. Bayesian estimation requires that the number of shocks must equal or exceed the number of observable variables used in the estimation. Since we have only two shocks in the No-KC and KC models, and three shocks in KC with LES model, then we are restricted to use at most two observable variables in the former two cases and no more than three observable variables in the latter case. Thus, we use the real growth rate of consumption (g_c) and hours of work (g_n) as the observable variables in the above three cases. This also allows us to have a fair comparison of the three models. Later, we add an additional observable variable, the real growth rate of output (g_y) to the estimation of KC with LES model and call it KC with LES^* specification.

In both the No-KC and the KC models, more than 95% of the observed fluctuations in g_c are explained by the preference shock. Also, about two-thirds of the observed fluctuations in g_n are explained by the stationary TFP shock and one-third by the preference shock. This contrasts sharply with the posterior variance decomposition associated with the knowledge capital with learning efficiency shock (KC with LES) model, Table 9. The importance of the stationary TFP shock in explaining the movement of hours worked falls from over 60% to a maximum of 15%. Similarly, the growth rate of consumption is no longer fully explained by preference shocks: falling from almost 100% to around 50%. The gap in explanatory power of the traditional shocks is filled by the learning efficiency shock which explains upto 46.40% and 59.78% of the observed fluctuations in g_c and g_n , respectively. Once again our earlier discussion about how learning efficiency shocks act as shift factors in the labour market is relevant for understanding these results as is the discussion regarding knowledge capital as an endogenous source of productivity variation. For completeness and as further evidence of the fit of the model we report plots of the data and predicted series for the KC with LES model in Figures 2 and 3. As is evident from these figures, the predicted and the actual series are too close to be distinguished from each other.

Our results regarding the importance of learning efficiency shocks still hold when using g_y as an additional observable variable (see Table 10). The presence of the growth rate of output resurrects a role for stationary TFP shocks in explaining consumption growth to about 25%, an increase that comes at the expense of preference shocks for the most part. The bulk of the movements in output are explained by stationary TFP shocks. Given these results and the importance of learning efficiency shocks in explaining movements in hours, we carry out a parallel set of estimation exercises using g_y instead of g_n as an observable variable in the No-KC, KC, and KC with LES models (see Tables A1 to A7). These results are reported in the appendix. The main result worth highlighting is that learning efficiency shocks continue to play a large role in explaining business cycle fluctuations. As before, the log data density rises in the presence of these shocks. A comparison across the results suggest that models without knowledge capital and learning efficiency shocks have a much harder time accounting for the joint behaviour of consumption and hours than in explaining consumption and output.

4 Conclusion

We introduce learning efficiency shocks into a DSGE model with knowledge capital. These shocks have two important effects on the model. First, they act as powerful shifters of labour demand, inducing changes in hours worked not captured by the standard calculus of the static marginal product of labour. A rise in learning efficiency implies it is a good time to learn so firms “invest” in knowledge capital by hiring more labour than is justified by the additional current output produced by these hours. The extra knowledge capital, acts as an endogenous source of future productivity increases which induce further increases in observed inputs and output. We estimate three versions of our model. The No-KC model has neither knowledge capital nor shocks to learning efficiency. The KC model incorporates knowledge capital while

the KC with LES model has both. Using Bayesian inference procedures and US real aggregate data on the growth rate of consumption, hours worked, and output, we find evidence for the presence of knowledge capital as well as learning efficiency shocks. Our results confirm that learning efficiency shocks are an important source of observed fluctuations in the growth rate of consumption, hours worked, and output. They are particularly helpful in accounting for the joint behaviour of consumption and hours, a problem that has plagued the business cycle literature from the start.

Table 2: No-KC model: Parameter and Shock Estimates

Parameter	<i>Prior distribution</i>			<i>Estimated posterior</i>		<i>Posterior distribution</i>			
	type	mean	st. error	mode	st. error	10%	median	90%	mean
α	beta	0.6000	0.1000	0.2081	0.0439	0.1440	0.2130	0.2803	0.2148
ρ_A	beta	0.8500	0.0500	0.9407	0.0112	0.9208	0.9397	0.9575	0.9392
ρ_θ	beta	0.8500	0.0500	0.9643	0.0043	0.9568	0.9637	0.9705	0.9635
σ_A	inverse gamma	0.0100	∞	0.0077	0.0004	0.0070	0.0077	0.0084	0.0077
σ_θ	inverse gamma	0.0100	∞	0.0129	0.0006	0.0120	0.0130	0.0140	0.0130

Notes: Log data density is 1779.778.

Table 3: KC model: Parameter and Shock Estimates

Parameter	<i>Prior distribution</i>			<i>Estimated posterior</i>		<i>Posterior distribution</i>			
	type	mean	st. error	mode	st. error	10%	median	90%	mean
α	beta	0.6000	0.1000	0.2236	0.0394	0.1609	0.2234	0.2888	0.2246
ε	beta	0.2500	0.1000	0.2392	0.0493	0.1581	0.2378	0.3184	0.2390
γ	beta	0.8000	0.1000	0.4885	0.0800	0.3652	0.4943	0.6255	0.4947
ρ_A	beta	0.8500	0.0500	0.9142	0.0132	0.8912	0.9135	0.9345	0.9129
ρ_θ	beta	0.8500	0.0500	0.9472	0.0058	0.9374	0.9470	0.9563	0.9468
σ_A	inverse gamma	0.0100	∞	0.0079	0.0004	0.0072	0.0080	0.0089	0.0080
σ_θ	inverse gamma	0.0100	∞	0.0133	0.0005	0.0124	0.0134	0.0144	0.0134

Notes: Log data density is 1788.561.

Table 4: KC with LES model: Parameter and Shock Estimates

Parameter	<i>Prior distribution</i>			<i>Estimated posterior</i>		<i>Posterior distribution</i>			
	type	mean	st. error	mode	st. error	10%	median	90%	mean
α	beta	0.6000	0.1000	0.2988	0.0462	0.2388	0.3098	0.3724	0.3080
ε	beta	0.2500	0.1000	0.1945	0.0324	0.1460	0.1996	0.2537	0.2010
γ	beta	0.8000	0.1000	0.9339	0.0213	0.8694	0.9245	0.9639	0.9150
ρ_A	beta	0.8500	0.0500	0.8641	0.0475	0.7794	0.8561	0.9325	0.8525
ρ_B	beta	0.8500	0.0500	0.8225	0.0459	0.7559	0.8342	0.9159	0.8350
ρ_θ	beta	0.8500	0.0500	0.9631	0.0045	0.9538	0.9616	0.9689	0.9614
σ_A	inverse gamma	0.0100	∞	0.0029	0.0005	0.0021	0.0029	0.0039	0.0030
σ_B	inverse gamma	0.0100	∞	0.0288	0.0072	0.0181	0.0276	0.0390	0.0284
σ_θ	inverse gamma	0.0100	∞	0.0106	0.0005	0.0098	0.0106	0.0115	0.0106

Notes: Log data density is 1850.980. Two chains of 300,000 draws are generated using a Random walk Metropolis algorithm. We discard the first 100,000 draws.

Table 5: KC with LES^* spec: Parameter and Shock Estimates

Parameter	<i>Prior distribution</i>			<i>Estimated posterior</i>		<i>Posterior distribution</i>			
	type	mean	st. error	mode	st. error	10%	median	90%	mean
α	beta	0.6000	0.1000	0.3985	0.0381	0.3295	0.3925	0.4572	0.3921
ε	beta	0.2500	0.1000	0.2713	0.0349	0.2189	0.2764	0.3319	0.2760
γ	beta	0.8000	0.1000	0.9823	0.0088	0.9609	0.9802	0.9944	0.9777
ρ_A	beta	0.8500	0.0500	0.9807	0.0057	0.9691	0.9791	0.9887	0.9788
ρ_B	beta	0.8500	0.0500	0.9771	0.0094	0.9615	0.9778	0.9924	0.9764
ρ_θ	beta	0.8500	0.0500	0.9808	0.0038	0.9749	0.9815	0.9874	0.9813
σ_A	inverse gamma	0.0100	∞	0.0075	0.0003	0.0070	0.0076	0.0082	0.0076
σ_B	inverse gamma	0.0100	∞	0.0028	0.0003	0.0023	0.0029	0.0035	0.0029
σ_θ	inverse gamma	0.0100	∞	0.0101	0.0005	0.0091	0.0100	0.0109	0.0100

Notes: Log data density is 2757.629. In addition to g_c and g_n , we use g_y as an observable variable in estimation.

Table 6: Summary of Log data densities

	Log data density
No-KC model	1779.778
KC model	1788.561
KC with LES model	1850.980
KC with LES^* spec	2757.629

Note: KC with LES^* spec.: in addition to g_c and g_n , we use g_y as an observable variable in estimation.

Table 7: No-KC model: Variance Decomposition Based on the Estimated Parameter Values

	Horizon	A	θ
Observable variable g_c	1	1.46	98.54
	2	1.54	98.46
	4	1.68	98.32
	10	1.90	98.10
	40	2.01	97.99
	∞	2.08	97.92
Observable variable g_n	1	71.30	28.70
	2	71.37	28.63
	4	71.47	28.53
	10	71.66	28.34
	40	71.74	28.26
	∞	71.73	28.27

Table 8: KC model: Variance Decomposition Based on the Estimated Parameter Values

	Horizon	A	θ
Observable variable g_c	1	3.12	96.88
	2	3.20	96.80
	4	3.33	96.67
	10	3.48	96.52
	40	3.52	96.48
	∞	3.61	96.39
Observable variable g_n	1	65.91	34.09
	2	65.85	34.15
	4	66.01	33.99
	10	66.55	33.45
	40	66.63	33.37
	∞	66.61	33.39

Table 9: KC with LES model: Variance Decomposition Based on the Estimated Parameter Values

	Horizon	A	B	θ
Observable variable g_c	1	0.27	41.94	57.79
	2	0.28	41.90	57.82
	4	0.29	42.07	57.63
	10	0.29	43.58	56.13
	40	0.29	44.96	54.76
	∞	0.29	46.40	53.31
Observable variable g_n	1	15.46	0.14	84.40
	2	9.85	37.50	52.65
	4	7.67	52.44	39.89
	10	7.39	55.18	37.43
	40	6.64	59.69	33.66
	∞	6.62	59.78	33.61

Table 10: KC with LES^* spec: Variance Decomposition Based on the Estimated Parameter Values

	Horizon	A	B	θ
Observable variable g_c	1	25.47	38.45	36.08
	2	25.55	38.40	36.05
	4	25.68	38.32	36.00
	10	25.82	38.34	35.84
	40	24.84	40.77	34.38
	∞	24.26	42.71	33.03
Observable variable g_n	1	19.01	18.97	62.02
	2	18.97	19.30	61.72
	4	18.91	19.84	61.24
	10	18.83	20.76	60.41
	40	18.86	21.19	59.95
	∞	18.80	21.32	59.87
Observable variable g_y	1	87.83	2.85	9.32
	2	87.24	3.50	9.26
	4	86.14	4.71	9.14
	10	83.46	7.68	8.86
	40	78.64	13.03	8.33
	∞	76.99	14.87	8.15

Note: KC with LES^* specification: in addition to g_c and g_n , we use g_y as an observable variable in estimation.

Figure 2: Quarter-over-quarter consumption growth in the data and in the model with learning-efficiency shocks.

Figure 3: Quarter-over-quarter growth rate of hours in the data and in the model with learning-efficiency shocks.

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A Appendix

Table A1: No-KC model: Parameter and Shock Estimates

Parameter	<i>Prior distribution</i>			<i>Estimated posterior</i>		<i>Posterior distribution</i>			
	type	mean	st. error	mode	st. error	10%	median	90%	mean
α	beta	0.6000	0.1000	0.5399	0.0403	0.4655	0.5327	0.5973	0.5311
ρ_A	beta	0.8500	0.0500	0.9734	0.0075	0.9599	0.9730	0.9845	0.9724
ρ_θ	beta	0.8500	0.0500	0.9249	0.0174	0.8969	0.9258	0.9529	0.9250
σ_A	inverse gamma	0.0100	∞	0.0056	0.0003	0.0051	0.0056	0.0063	0.0057
σ_θ	inverse gamma	0.0100	∞	0.0103	0.0007	0.0092	0.0104	0.0117	0.0104

Notes: Log data density is 1817.996.

Table A2: KC model: Parameter and Shock Estimates

Parameter	<i>Prior distribution</i>			<i>Estimated posterior</i>		<i>Posterior distribution</i>			
	type	mean	st. error	mode	st. error	10%	median	90%	mean
α	beta	0.6000	0.1000	0.3639	0.0498	0.2759	0.3564	0.4405	0.3567
ε	beta	0.2500	0.1000	0.2984	0.0530	0.2138	0.2980	0.3897	0.2994
γ	beta	0.8000	0.1000	0.4514	0.0920	0.3196	0.4650	0.6132	0.4664
ρ_A	beta	0.8500	0.0500	0.9560	0.0121	0.9353	0.9558	0.9734	0.9545
ρ_θ	beta	0.8500	0.0500	0.9532	0.0153	0.9289	0.9523	0.9758	0.9513
σ_A	inverse gamma	0.0100	∞	0.0064	0.0004	0.0057	0.0064	0.0071	0.0065
σ_θ	inverse gamma	0.0100	∞	0.0115	0.0011	0.0098	0.0115	0.0134	0.0116

Notes: Log data density is 1833.224.

Table A3: KC with LES model: Parameter and Shock Estimates

Parameter	<i>Prior distribution</i>			<i>Estimated posterior</i>		<i>Posterior distribution</i>			
	type	mean	st. error	mode	st. error	10%	median	90%	mean
α	beta	0.6000	0.1000	0.4993	0.0512	0.3655	0.4759	0.5658	0.4687
ε	beta	0.2500	0.1000	0.1809	0.0503	0.1184	0.2072	0.3073	0.2114
γ	beta	0.8000	0.1000	0.6449	0.1231	0.4810	0.7000	0.9636	0.7069
ρ_A	beta	0.8500	0.0500	0.8858	0.0540	0.8234	0.9094	0.9758	0.9006
ρ_B	beta	0.8500	0.0500	0.9487	0.0195	0.7752	0.9097	0.9724	0.8862
ρ_θ	beta	0.8500	0.0500	0.9608	0.0093	0.9403	0.9593	0.9773	0.9578
σ_A	inverse gamma	0.0100	∞	0.0037	0.0008	0.0028	0.0042	0.0061	0.0044
σ_B	inverse gamma	0.0100	∞	0.0133	0.0048	0.0038	0.0116	0.0185	0.0121
σ_θ	inverse gamma	0.0100	∞	0.0116	0.0009	0.0100	0.0116	0.0134	0.0117

Notes: Log data density is 1835.872.

Table A4: Summary of Log data densities: Using g_y instead of g_n as an observable variable

	Log data density
No-KC model	1817.996
KC model	1833.224
KC with LES model	1835.872
KC with LES^* model	2757.629

Note: KC with LES^* model: in addition to g_c and g_y , we use g_n as an observable variable in estimation.

Table A5: No-KC model: Variance Decomposition Based on the Estimated Parameter Values

	Horizon	A	θ
Observable variable g_c	1	9.88	90.12
	2	9.90	90.10
	4	9.93	90.07
	10	9.99	90.01
	40	10.01	89.99
	∞	10.26	89.74
Observable variable g_y	1	90.07	9.93
	2	90.00	10.00
	4	89.89	10.11
	10	89.73	10.27
	40	89.68	10.32
	∞	89.70	10.30

Table A6: KC model: Variance Decomposition Based on the Estimated Parameter Values

	Horizon	A	θ
Observable variable g_c	1	12.72	87.28
	2	12.78	87.22
	4	12.91	87.09
	10	13.04	86.96
	40	13.14	86.86
	∞	13.34	86.66
Observable variable g_y	1	91.66	8.34
	2	89.75	10.25
	4	89.08	10.92
	10	89.03	10.97
	40	88.76	11.24
	∞	88.74	11.26

Table A7: KC with LES model: Variance Decomposition Based on the Estimated Parameter Values

	Horizon	A	B	θ
Observable variable g_c	1	2.10	8.95	88.95
	2	2.14	8.95	88.91
	4	2.18	9.03	88.79
	10	2.19	9.36	88.45
	40	2.26	9.57	88.17
	∞	2.27	9.79	87.94
Observable variable g_y	1	66.44	1.22	32.34
	2	52.52	21.25	26.23
	4	46.04	30.76	23.20
	10	45.54	31.98	22.49
	40	44.58	33.14	22.28
	∞	44.49	33.21	22.30

Figure A1: Dynamic Responses to TFP Shock: All responses are displayed relative to the size of the shock.

Figure A2: Dynamic Responses to Preference Shock: All responses are displayed relative to the size of the shock.