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Investing in Health: A Macroeconomic Exploration of Short-Run and Long-Run Trade-Offs

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Abstract

This paper aims to unravel the competing effects of the health investment. It explores, both analytically and numerically, the equilibrium shift and transitional dynamics after a one-time policy of health investment. We find that such a policy improves health status in the long run, but harms economic growth in both short and long term. The relative sizes of these competing effects depend on the specific health parameters. Within the plausible range for the value of health relative to consumption, households gain welfare in the long run as long as the effectiveness of labor in health production is large. The expanded health sector policy makes households worse off only if labor is rather unproductive in producing health and households value health relatively little. The findings challenge the policy recommendations of the World Bank (1993) and World Health Organization (2001) in that good health increases neither the productivity of workers nor the economic growth rate. It is hoped that the relative simplicity of our model, compared to the existing theoretical literature, can help close the gap between formal academic work on this topic and actual debates among policy makers in both developed and developing countries.

JEL classification: E2; E6; O4; I1

Keywords: Health capital; Health investment; Endogenous growth; Dynamic system; Transitional dynamics

1 Introduction

Investing in health has been promoted worldwide for economic development. The World Bank (1993) (Abstract, p.17, p.51) states that “because good health increases the economic productivity of individuals and the economic growth rate of countries, investing in health is one means of accelerating development.” The World Health Organization (2001) (p.1–3) reports that “the linkages of health to... long-term economic growth are powerful,... the improvements in health would translate into higher incomes and higher economic growth.” But does investment in health necessarily cause economic growth and make people better off? And for all countries? Not necessarily. Investing in health involves trade-offs such as short-term losses in consumption and the associated loss in welfare versus longer-term enhanced health, which directly improves welfare and can potentially generate productivity gains that increase long-run consumption and welfare. This paper aims to address these questions by disentangling the competing effects.

The most direct effect of investing in health is positive if it makes people healthier, and hence better off. But investing in health requires that resources be reallocated from productive uses outside the health sector, such as in the manufacturing sector. Other things equal, in the short run, this reduces consumption, which harms welfare. In the longer run, however, things are not equal if healthier workers are more productive workers. This indirect effect has two implications. First, it tempers and can even reverse the reduction in manufacturing output. Second, it increases the incentive to invest in capital accumulation because more productive workers may raise the marginal product of physical capital. If so, households react by saving more, and the standard short-term-pain-but-long-term-gain result emerges. Increased saving means less consumption in the short run but more consumption in the long run. As a consequence, if the productivity-enhancing effect of health investment were big enough,

there would be a win-win outcome — both more health and more manufactured goods (hence higher economic growth) in the long run.

However, if the productivity-enhancing effect of better health were “small”, then this win-win outcome would not occur. The resulting reduction in total output would lead to a decrease in the marginal product of physical capital, less household saving and more consumption initially. Less short-term pain is incurred, but no long-term gain arises. In this case, the analysis does not support investment in health as a mechanism for raising economic growth, though it would still be possible to support it if it led to sufficient welfare gains. The net welfare effect depends on the relative size of the direct effect of health on welfare and the loss in welfare from less consumption. It is critical to investigate these dynamic effects of investments in health.

The available theoretical literature that attempts to unravel such trade-offs is incomplete. The seminal work, Grossman (1972), is the first to formally model health as an investment good. However, what Grossman develops is a partial equilibrium analysis of optimal investment on the part of one individual, which does not consider general equilibrium effects such as the productivity-enhancing effect of health on manufacturing workers from a broader view of an entire economy.

Later literature extends this “health capital” notion in the setting of general equilibrium. However, some studies analyze only the equilibrium effects caused by investment in health and do not consider the transitional dynamics between equilibria. For example, Van Zon and Muysken (2001) recognize that health sector is a labor-intensive sector and analyze the trade-off of health investment in the form of labor allocation between the manufacturing and the health sector. But they do not provide an analytical solution. Van Zon and Muysken (2005) do present an analytical solution in an extended model, and assess these competing effects, but only at the long-run equilibrium. Their numerical results indicate that an investment in care services provided by health labor has a positive welfare effect, raising life expectancy, but

it negatively affects the output of the economy. However, these are the steady-state effects; the transitional dynamics (both analytical and numerical) between equilibria are still missing.

There is no agreement concerning the weight on health that is appropriate, relative to consumption, in the household utility function. For example, Hall and Jones (2007) construct an age-specific overlapping generations model where the households' choice is to increase spending on health, which extends life and enhances utility. They assume that the marginal utility of life extension does not decline. As the health spending share of GDP grows along with income, they project that the optimal share in the United States is likely to exceed 30 percent by 2050. The reciprocal of health status enters the utility function as the rate of time preference in their basic model, which allows an interaction between health and consumption in utility, but in the full dynamic version of their model, they specify that health enters the utility function as a separate variable. This requires that they assign a utility weight to health relative to consumption, which they do by drawing on an observation of Nordhaus (2002) regarding the historical value of increases in income and life expectancy in the US. But there is still considerable uncertainty about how households value health relative to consumption. Hence, it is important to assess how sensitive their conclusions are to the assumed weights in the utility function.

This paper contributes in several ways to the theoretical analysis of health investment in a general equilibrium framework. In a two-sector endogenous AK growth model, it specifies health and non-health production functions and includes health in households' utility function. It explores, both analytically and numerically, the equilibrium shift and transitional dynamics after a once-for-all policy of investment in health (modeled as a reallocation of labor from the non-health to the health sector). The numerically calibrated version of the model indicates for how long the pain of consumption loss lasts, and determines whether these losses are compensated by the

long-term gains in health, welfare, and perhaps productivity.

Unlike most of the literature which assumes private financing of health care, our analysis assumes public financing of the health investment, which accords with the institutional reality that publicly financed health care predominates internationally. Second, the model provides a sensitivity test for specifications of household utility. For one thing, our model does not rely on a strong interaction between health and consumption as in Hall and Jones' (2007) basic model. For another, our model assumes Ricardian equivalence (no potential conflicts between generations) — a further departure from Hall and Jones (2007). Third, while our model is simpler than the existing literature, it embodies the modern macroeconomic requirement of internal consistency and intertemporal optimization on the part of economic agents, and yet captures the essential trade-offs in a way that is more accessible to policy makers than is the existing theoretical literature. It is hoped that the relative simplicity of our model contributes to the policy debate in countries with a publically-financed health sector such as Canada, the United Kingdom, and the Scandinavian countries. Fourth, we conduct alternative calibrations using different parameter values in the entire range of the productivity-enhancing effect of health. The small value, in particular, is relevant for developed countries since the aged population who consume much of the health services may well have retired from the workforce. Thus, the results allow us to address the challenges faced by developed countries with aging populations.

Below section 2 presents a two-sector endogenous AK growth model. Sections 3 and 4 introduce a policy shock that diverts labor from the manufacturing to the health sector, analyze the resulting equilibrium shift and the transitional dynamics between the equilibria. Section 5 calibrates the model. Section 6 reports the simulation results and some sensitivity tests. Section 7 concludes and discusses.

2 A Two-Sector Model of Health Capital in an Endogenous AK Growth Model

The model economy involves two sectors: the health sector and the manufacturing sector. In the health sector, we assume that the production function has only one input, health labor (L), which is measured as a fraction of the total population (normalized to one); that is,

$$\dot{H} = \phi L^\gamma - \delta H. \quad (1)$$

$\dot{H}$ refers to the change in health status with respect to time. This production function involves positive but diminishing returns: $\phi > 0$ and $0 < \gamma < 1$. For simplicity, the depreciation rate of health status (δ) is assumed to be the same as that of physical capital ($0 < \delta < 1$) as is commonly done in the literature (e.g., Ried (1996); Halliday *et al.* (2014)).

In the non-health (manufacturing) sector, we assume individual firms are identical. In traditional growth theory, individual firms' Cobb-Douglas production is assumed to have diminishing return to physical capital in manufacturing production. But in the new growth theory, productivity is endogenously enhanced by investment in human capital in the form of knowledge and, as in this paper, health. The diminishing-return assumption holds for the individual firms' production function but not for the aggregate production function. To achieve this set of properties, the growth literature maintains a Cobb-Douglas production function with the diminishing return for individual firms (equation (2)), while assuming that the productivity of each worker is proportional to the level of human capital (knowledge), which — in turn — is proportional (θ) to the *aggregate* physical capital stock (and health status, equation (3)). For the whole economy, then, there is a linear production function with a constant return for the physical capital input for the society (equation (4)). In

particular, the individual (denoted with the “i” subscript) firms’ production function has the Cobb-Douglas form in which the effective labor input ($q(1-L)$). The extension to the standard Romer (1990)’s AK growth model that is involved here is that worker productivity is affected by both education and health ($\beta > 0$); that is,

$$Y_i = K_i^\alpha (q(1 - L_i))^{1-\alpha}, \quad (2)$$

$$q = \theta K H^\beta. \quad (3)$$

We assume that health raises labor productivity but with diminishing returns ($0 < \beta < 1$). The only restriction for the scale parameter θ is that it be positive. Equations (2) and (3) can be combined to yield, at the aggregate level, the society-level production function in the manufacturing sector

$$Y = AK, \quad (4)$$

where

$$A = \theta^{1-\alpha} H^{\beta(1-\alpha)} (1 - L)^{1-\alpha}. \quad (5)$$

Firms in the manufacturing sector pay the interest rate (r) to rent physical capital and the wage rate (w) to rent labor. They hire each factor up to the point that the marginal product equals the rental price; that is,

$$\frac{\alpha Y}{K} = \alpha A = r + \delta, \quad (6)$$

which is the firm’s optimal hiring rule for physical capital; and

$$\frac{(1 - \alpha)Y}{1 - L} = w, \quad (7)$$

which is the optimal hiring rule for labor.

Next, we define the economy's resource constraint

$$Y = C + \dot{K} + \delta K + G = w(1 - L) + rK + \delta K, \quad (8)$$

where G is government spending on non-health programs whose ratio to the total manufacturing output, $z = G/Y$, is assumed to be constant in our policy analysis. The wage rate (w) for health and non-health labor is assumed to be the same. The interest rate (r) is the rate of return on capital. The constraint says that the sum of labor and capital income equals the sum of the households' spending on consumption and investment, and the government's spending on non-health programs.

Although the optimal choice of investing in health is a mixed decision by both households and the government, countries with a dominant publically-financed health sector such as Canada and the U.K. can control the health investment through both financing policies and controlling the admissions of domestic medical schools and the immigration of international medical graduates. Moreover, the health sector is a labor-intensive service sector. Thus, we assume that the choice of optimal health investment is made solely by the government. The government's budget constraint is

$$G + wL = \tau(w + rK), \quad (9)$$

which states that the government collects taxes imposed on both labor and capital income (net of depreciation expenses) at the same tax rate (τ), and uses this revenue to pay the wages of the health labor, and the non-health governmental programs (G). Note that identity (9) can be rewritten using the optimal hiring rules as

$$z + \frac{(1 - \alpha)L}{1 - L} = \left(\frac{\tau}{1 - \tau}\right)\left(1 - \frac{\delta}{A} - z\right). \quad (10)$$

Households maximize utility, which involves additive terms of consumption and health status in a logarithm form and a constant rate of time preference (ρ)

$$U = \int_0^\infty e^{-\rho t} (\ln C_t + \xi \ln H_t) dt, \quad (11)$$

subject to the constraint that total expenditures equal total income; that is,

$$C + \dot{K} = (1 - \tau)(rK + w). \quad (12)$$

This optimization yields the familiar Ramsey (1928) consumption function

$$\frac{\dot{C}}{C} = r(1 - \tau) - \rho. \quad (13)$$

We use household utility as our measure of welfare in our policy analysis.

3 The Pre-Policy Equilibrium and the Shift to the Post-Policy Equilibrium

We explore the pre-policy equilibrium of the economy. Then we introduce an exogenous health labor policy that reallocates some of the nation's workforce from the manufacturing sector to the health sector, doubling health labor in a once-for-all fashion.

3.1 Pre-Policy Equilibrium

To identify the pre-policy equilibrium, first we define the ratio of consumption to physical capital, $x = C/K$. We are interested in both consumption and the physical capital stock independently. But since both of them rise to infinity as time proceeds,

only the ratio of the two has a fixed value to which the dynamic system of the economy can converge.

We summarize the model as a system of four equations. First, we have

$$\frac{\dot{x}}{x} = [\alpha(1 - \tau) - (1 - z)]A - \rho + x + \tau\delta, \quad (14)$$

which is obtained by substituting equation (13) and the left-hand side of equation (8) into the time derivative of x . The other three equations are restatements of equations already presented

$$\frac{\dot{H}}{H} = \phi L^\gamma H^{-1} - \delta, \quad (1)$$

$$A = [\theta(1 - L)]^{1-\alpha} H^{\beta(1-\alpha)}, \quad (5)$$

$$\tau = \frac{z + (\frac{1-\alpha}{1-L})L}{1 - \frac{\delta}{A} + (\frac{1-\alpha}{1-L})L}. \quad (10)$$

These four equations, (14, 1, 5, and 10), involve five endogenous variables — the time rate of change in H and x , and the current values of x , A , and τ . Because the two ordinary differential equations (1) and (14) involving H and x generate a two-dimensional dynamic system that represents the core of this economy, we focus on these two equations to find the equilibrium. At each point in time, when the values of A , τ , and H are given, these two equations determine x , $\dot{x}$, and $\dot{H}$, since x is a jump variable, while H is not. The reason for this difference is that part of x — consumption — is determined by the households' *forward-looking* consumption plan; whereas the change in health status follows from a historically based (*backward-looking*) accumulation identity. Thus, while x can jump at a point in time, H cannot.

To close the model, we need a fifth restriction together with the four equations above to solve for the five endogenous variables. The standard procedure is to assume whatever it takes to ensure unique convergence to a full equilibrium (that is, to apply Samuelson (1941)'s Correspondence Principle). In this case, we need to check whether

the model involves a saddle path; and if so, to assume that the initial value of x is determined by requiring that the economy be on that saddle path.

Now we verify whether such a saddle equilibrium exists, then specify the saddle-path equation. We examine the equilibrium and its neighborhood by linearization, as widely done in the literature (Scarth, 2014). Taking a linear approximation (the total differential) of equations (1) and (14) gives

$$\begin{bmatrix} d\dot{H} \\ d\dot{x} \end{bmatrix} = B \begin{bmatrix} dH \\ dx \end{bmatrix} + \begin{bmatrix} \phi\gamma L^{\gamma-1} \\ \left((\alpha(1-\tau) - (1-z) + rq_2) \left(\frac{\alpha-1}{1-L} \right) A - rq_1 \right) x \end{bmatrix} dL, \quad (15)$$

where

$$B = \begin{bmatrix} -\delta & 0 \\ \frac{(\alpha(1-\tau) - (1-z) + rq_2) \beta(1-\alpha) Ax}{H} & x \end{bmatrix}, \quad (16)$$

$$q_1 = \frac{\left(\frac{1-\alpha}{1-L} \right) \left(1 + \frac{L-\tau}{1-\tau} \right)}{\frac{1-\alpha}{1-L} + \frac{r}{A}},$$

$$q_2 = \frac{\frac{\tau\delta}{A^2}}{\frac{1-\alpha}{1-L} + \frac{r}{A}}.$$

The determinant of matrix B is the product of the two eigenvalues. Since this expression, $-\delta x$, is negative, one eigenvalue must be positive and the other negative. This verifies that there exists a saddle equilibrium for the system. As noted, this is necessary for unique convergence since there is one sticky variable, H , and one jump variable, x . Moreover, because the negative real eigenvalue (here it is $-\delta$) is the adjustment speed of the dynamic system (Smale *et al.*, 2012), we define $v = -\delta$ as the velocity of our system adjusting along the saddle path. Given the linear approximation, all endogenous variables move with this adjustment speed between the initial and final equilibria.

3.2 Exogenous Investment in Health Sector

We now introduce the exogenous health labor policy that reallocates the manufacturing labor to the health sector. We assume for simplicity that the health labor policy is a once-for-all event that households and firms do not anticipate. x jumps to put the economy on the new saddle path immediately after the policy shock. Then, as time passes following this initial change, the economy tracks along the new saddle path. As noted in the introduction, one possibility is that the short-term pain (lower consumption initially) could be followed by a long-term gain (higher consumption in the post-policy full equilibrium). To establish what outcome is likely, we must examine a calibrated version of the model.

Given parameter values and initial values of τ , L and H , we can derive the effects of the health labor policy on the full-equilibrium values of health status (dH^*/dL) and the ratio of consumption to physical capital (dx^*/dL). To do so, we set $\dot{H}$ and $\dot{x}$ to zero. Since $d\dot{H}$ and $d\dot{x}$ are defined as $(\dot{H} - \dot{H}^*)$ and $(\dot{x} - \dot{x}^*)$, and since $\dot{H}^* = \dot{x}^* = 0$, we set $d\dot{H}$ and $d\dot{x}$ to zero in equation (15) to get

$$\begin{bmatrix} \delta & 0 \\ -\frac{(\alpha(1-\tau)-(1-z)+rq_2)\beta(1-\alpha)Ax}{H} & -x \end{bmatrix} \begin{bmatrix} dH^* \\ dx^* \end{bmatrix} = \begin{bmatrix} \phi\gamma L^{\gamma-1} \\ \left((\alpha(1-\tau)-(1-z)+rq_2)\left(\frac{\alpha-1}{1-L}\right)A - rq_1 \right)x \end{bmatrix} dL. \quad (17)$$

Denote the left-hand-side matrix (which is simply the negative of matrix B) as B' . Define two matrices that are associated with B' — B_H and B_x where the column in front of dL respectively replaces the first and second column of B' .

By Cramer's rule using the determinants of these matrices, the full-equilibrium shifts dH^* and dx^* following a change in the size of the health sector workforce dL are $dH^*/dL = |B_H|/|B'|$ and $dx^*/dL = |B_x|/|B'|$.

Further, by assumptions in section 2, $\phi > 0$, $0 < \delta < 1$, $0 < \gamma < 1$, and $0 < L < 1$,

we derive that $dH^*/dL = \phi\gamma L^{\gamma-1}/\delta > 0$. This means that an increase in the health labor raises the equilibrium value of health status. However, dx^*/dL has a very complicated expression, and its sign is indeterminate, hence the need for calibration.

Given $dH^* = H^* - H_0$ and $dH^* = (|B_H|/|B'|) dL$, the post-policy equilibrium value of health status is

$$H^* = H_0 + dH^* = H_0 + \frac{|B_H|}{|B'|} dL, \quad (18)$$

where H_0 is the initial value of the health status before policy.

Similarly, the post-policy equilibrium value of x is

$$x^* = x_0 + dx^* = x_0 + \frac{|B_x|}{|B'|} dL, \quad (19)$$

where x_0 is the initial value of x before policy (the pre-policy equilibrium value).

We have found the pre-policy and the post-policy equilibria. However, knowing only the equilibria before and after the policy is insufficient, given our goal to compare the short- and long-term pain and gain throughout the time horizon following the policy. Thus, we derive the transitional dynamics from the pre-policy to the post-policy full equilibria at each point in time for variables of interest: health status (H_t), the ratio of consumption to physical capital (x_t), the physical capital (K_t), consumption (C_t), and discounted summed utility (U_t). The details are in the Appendix.

4 Calibration of Pre-Policy Equilibrium and Sensitivity Tests of Health Parameters

To generate numerical simulations, we need to calibrate the model—in particular, the pre-policy equilibrium—and then examine the equilibrium shift caused by the health

labor policy and conduct sensitivity tests. We choose plausible values of primitive parameters and pre-determined variables by two criteria. First, they must satisfy all equations in section 2 when the economy is in its original full equilibrium (recall $\dot{H} = 0$, $\dot{x} = 0$, and $\dot{K}/K = \dot{C}/C = g$). Second, broadly speaking, they should match real world observations. Table 1 shows all but three of the primitive parameters and pre-determined variables at the pre-policy equilibrium, denoted with a “0” subscript. These chosen values are consistent with a large amount of applied macroeconomic literature. For example, the capital-output ratio, the annual depreciation rate and real interest rate are 3.3, 4% and 6% respectively; the capital’s share of income is 1/3; the annual real economic growth rate is 2%; and the income tax rate is 0.3. These parameters imply very sensible values for the shares of GDP represented by consumption, investment and non-health government spending, and an entirely standard value for the rate of time preference (see Table 1).

We normalize the size of the entire workforce to 1. For Canada, health labor accounts for 5% of the total workforce (Statistics Canada, 2014), so we let the initial L value (L_0) be 0.05. The health investment policy is to divert labor from manufacturing to double the health sector’s share from 0.05 to 0.10, that is, $dL = 0.05$. We normalize health status as an index with a starting value (H_0) of 1. There are many measurements or proxies of health level in the literature (e.g., life expectancy at birth or any other specific age, infant mortality rate). By indexing health level, we can obtain a much more general measurement, which can then be transformed back to the level of any specific health measure.

We now derive the three health-sector parameters (β , ξ , and γ) that are less well determined since they are less often reported in the literature. Tompa (2002) systematically reviews the empirical evidence about how much the increase in labor productivity is due to workers being healthier (e.g., Knowles and Owen (1997); Bloom *et al.* (2004)). Unfortunately, these estimates vary considerably across studies

Table 1: Calibrate the baseline equilibrium

δ	depreciation rate	0.04
g_0	annual economic growth rate	0.02
Y_0	total output	2
I_0	ratio of investment to total output	0.2
K_0	physical capital	6.667
A_0	productivity of non-health goods' production	0.3
α	return to physical capital in non-health goods' production	1/3
L_0	fraction of total labor force in health sector	0.05
r_0	real interest rate excluding the depreciation rate	0.06
w_0	wage	1.404
τ_0	income tax rate	0.3
G_0	governmental spending on non-health sector	0.471
ρ	time preference rate	0.022
H_0	index of health level	1
C_0	consumption of non-health goods	1.129
x_0	ratio of consumption to physical capital	16.94%

Notes: Pre-determined variable at the pre-policy equilibrium is denoted with a “0” subscript.

depending on the specific data, methods, and health measures used. Such a variety can be rationalized by one argument in Barro and Sala-i-Martin (1995) (p.432): “it is likely that life expectancy has such a strong, positive relation with growth because it proxies for features other than good health that reflect the desirable performance of a society. For example, higher life expectancy may go along with better work habits and a higher level of skills (for given measured values of per capita product and years of schooling).” Given this considerable uncertainty concerning the productivity-enhancing effect of health (β), and given the assumption of diminishing return $0 < \beta < 1$, we report three values within this range — “small” as 0.05 (which may better reflect the experience of developed countries since their aged population who need health services most may have retired from the labor force); “large” as 0.95 (in contrast, perhaps better representing the experience in developing countries); and

“moderate” as 0.50.

To choose a calibrated value for the relative weight of health in the utility function (ξ), we exploit two observations. Jones (2013) (p.1) reports that “during the twentieth century, life expectancy in the United States rose from less than 50 years to 77 years, while average incomes rose by about a factor of 7.” Jones also notes that Nordhaus (2002) estimates that individuals have evaluated both these favorable outcomes as generating about the same change in their utility. Now consider our instantaneous utility function in equation (11) which implies

$$\frac{dU}{U} = \frac{dC}{C} + \xi \frac{dH}{H}.$$

Set $dU/U = 0$, we have $dC/C = -\xi dH/H$. The two observations $dH/H = 1/2$ and $dC/C = 7$ implies that ξ is 14. We view the value of 14 as likely being at the high end of plausible values because it is based only on health changes that lengthen life. In reality, many health improvements produce improvements in quality of life while not extending it. Thus an alternative and smaller value, $\xi = 5$, is examined.

We construct two ways to pick a value for the parameter (γ) that defines how the investment in health labor increases overall health status. One way is via the connection between health labor L and expenditure on health labor as a percentage of GDP, which in our model is $Y + wL$. The proportion, denoted as E , of health expenditure to GDP is $E = wL/(Y + wL)$. Taking the differential implies

$$\frac{dE}{E} = \frac{Y}{Y + wL} \frac{dL}{L},$$

hence $dE/E = 0.588$ after substituting in values in Table 1. Assume further that the change in health is proportional to the change in health expenditure; that is,

$$\frac{dH}{H} = m \frac{dE}{E},$$

and then use actual data to estimate m . Between 1979 and 2007, for Canada (OECD (2015)) total health expenditure rose from 6.791% to 10.037% of GDP (so dE/E is 0.478), and life expectancy at birth rose from 75.1 to 80.7 years (so dH/H is 0.075). Then m is estimated to be 0.156. Combining the two relationships above gives

$$\frac{dH}{H} = \left(\frac{mY}{Y + wL} \right) \frac{dL}{L} = 0.09 \frac{dL}{L}.$$

Since the full-equilibrium version of equation (1) implies

$$\frac{dH}{H} = \gamma \frac{dL}{L}, \quad (20)$$

this reasoning and evidence justify our taking γ to be 0.09.

Another way to find the γ value follows from accepting both the specific model and its calibration contained in Van Zon and Muysken (2005). This strategy leads to our parameter γ being 0.03 (see Appendix). We simulate with both γ values.

Table 2 summarizes the two, referred as the “large” and “small” , sets of key health parameters. Given the literature from which the β values were taken we interpret the “large” set as more relevant for developing economies and the “small” for developed economies. The two scale parameters (ϕ and θ) are derived from these three parameters via equations (1) and (5) respectively.

Table 2: Range of health parameter values

	β	ξ	γ	ϕ	θ
Large	0.95	14	0.09	0.052	0.173
Small	0.05	5	0.03	0.044	0.173
Middle	0.50	14 or 5	0.09	0.052	0.173
	0.50	14 or 5	0.03	0.044	0.173

5 Simulation Results

We report the pre-policy equilibrium first, and then the equilibrium shifts and transitional dynamics in the short (one year), medium (ten years), and long (thirty-five years) run caused by the policy that reallocates labor from the manufacturing to the health sector, doubling the health labor from 5% to 10% of the nation's workforce. The reported results contain two types of comparisons — Pre-policy vs. Post-policy using the large parameter set, and Post-policy using large parameter set vs. Post-policy using small parameter set. We then report how the conclusions are sensitive to the three health parameters, and a stylized simulation suitable for developed countries such as Canada.

5.1 Pre-Policy Equilibrium

Table 3 shows the calibrated pre-policy equilibrium where the majority of the variables remain at their pre-policy initial values as in Table 1. While the ratio x_t is fixed, its denominator (K_t) and numerator (C_t) keep growing, starting at 6.667 and 1.129, reaching at 13.425 and 2.274 after thirty-five years.

Table 3: Pre-policy equilibrium

	Pre-policy equilibrium		Pre-policy dynamics				
	$t = 0$	$t = j$	$t = 1$	$t = 10$	$t = 35$	$t = 50$	$t = \infty$
L_t	5%	5%	5%	5%	5%	5%	5%
H_t	1	1	1	1	1	1	1
x_t	16.94%	16.94%	16.94%	16.94%	16.94%	16.94%	16.94%
A_t	0.3	0.3	0.3	0.3	0.3	0.3	0.3
τ_t	30%	30%	30%	30%	30%	30%	30%
r_t	6%	6%	6%	6%	6%	6%	6%
g_t	2%	2%	2%	2%	2%	2%	2%
K_t	6.667	6.667	6.801	8.143	13.425	18.122	∞
C_t	1.129	1.129	1.152	1.379	2.274	3.069	∞

Notes: j denotes jump, which means the initial value if a policy is implemented.

5.2 Post-Policy Equilibrium and Transitional Dynamics Using the Large Parameter Set

After introducing the policy that doubles the health labor from 5% to 10% of the labor force, the economy's equilibrium shifts from the pre-policy toward the post-policy equilibrium. Table 4 reports the transitional dynamics between the two equilibria using the large parameter set. H_t increases gradually, reaching 1.048 after thirty-five years, and eventually converging to 1.064. If the health measure were life expectancy at birth, this would imply that after doubling the health labor force, the households' life expectancy at birth increases from 75 to 78.6 years after thirty-five years of policy, and converges to 79.8 years.

Table 4: Equilibrium shift and transitional dynamics caused by the policy that real-locates labor from manufacturing to health sector to double health labor from 5% to 10% of the nation's workforce using the **large** parameter set

	Pre-policy equilibrium	Post-policy dynamics					
	$t = 0$	$t = j$	$t = 1$	$t = 10$	$t = 35$	$t = 50$	$t = \infty$
L_t	5%	10%	10%	10%	10%	10%	10%
H_t	1	1	1.003	1.021	1.048	1.056	1.064
x_t	16.94%	16.67%	16.69%	16.83%	17.04%	17.10%	17.17%
A_t	0.3	0.289	0.290	0.293	0.298	0.299	0.301
τ_t	30%	33.07%	33.07%	33.01%	32.93%	32.91%	32.88%
r_t	6%	5.65%	5.66%	5.78%	5.94%	5.98%	6.03%
g_t	2%	1.58%	1.59%	1.67%	1.78%	1.81%	1.85%
K_t	6.667	6.667	6.765	7.814	12.321	16.381	∞
C_t	1.129	1.111	1.129	1.315	2.100	2.801	∞

Notes: j denotes jump, which means the post-policy initial value.

The ratio of consumption to physical capital, x_t suddenly jumps down from 16.94% to 16.67% as the post-policy initial value, then slowly increases to 17.04% after thirty-five years and converges to 17.17%, higher than what occurs without the policy. Such a sudden drop comes from the households' forward-looking cut in consumption right

after the implementation of the policy. And the long-term increase in x_t may result from higher labor productivity (confirmed below) and hence more manufacturing output available for consumption.

Productivity, A_t immediately drops from 0.3 to 0.289 due to the sudden withdrawal of manufacturing labor. It then slightly increases throughout the time and converges to 0.301, negligibly higher than 0.300 without the policy. This suggests that when health parameters (in particular, the productivity-enhancing effect of health, β) is large, productivity increases though the increase is negligible in the long run.

The tax rate, τ_t immediately jumps up from 30% to 33.07% because the government needs to raise more tax revenue to finance the publicly-funded expansion of health labor. The tax rate then slightly decreases and converges to 32.88%, still higher than the pre-policy initial value 30%. This is because the policy is permanent.

The real interest rate, r_t immediately decreases from 6% to 5.65%. This outcome follows from the optimal hiring rule for physical capital — capital's rate of return falls because its productivity, A_t , falls. r_t then slowly goes back to 6.03% at the post-policy equilibrium, which corresponds to the slightly enhanced productivity A_t as 0.301 in the long term.

The growth rate, g_t falls considerably from 2% to 1.58% right after the policy is implemented, which results from the decrease in the pre-tax interest rate and increase in the income tax rate. Both these developments decrease the households' incentive to save. But g_t largely recovers to 1.81% after half of a century and eventually converges to 1.85%, though still lower than the pre-policy initial growth rate 2%. This is because, in the long term, the decrease in the interest rate and the increase in the tax rate are both less than in the short term. Recall that the productivity-enhancing effect of health (β) is 0.95. This result means that even if health brings a nearly constant return in labor productivity, investment in health still slows down economic development; although more than half of the slowdown in the short run is

recovered in the long run.

Physical capital, K_t rises from 6.667 to 12.321 after thirty-five years of introducing the policy. Comparing to its counterpart (13.425) without the policy in Table 3, physical capital accumulation is reduced by the policy in both the short and long run. Again, this follows from the decreased incentive to save, mostly due to the higher tax rate.

Consumption, C_t jumps slightly down from 1.129 to 1.111 on the new saddle path toward the post-policy equilibrium because of households' forward-looking behavior in the face of the unexpected implementation of the policy. It then increases to 2.100 after thirty-five years. Comparing to its counterparts (1.152 and 2.274) without the policy in Table 3, consumption is hurt by the policy in both the short and long term.

Relative to the pre-policy equilibrium, the percentage change of the discounted utility summed over time right after the policy is -13.12% , which is an immediate pain. This pain is quickly relieved since the percentage change becomes smaller at -0.39% after one year. It then reverses the sign and reaches $+54.09\%$ after ten years and $+73.10\%$ after thirty-five years. Hence, under the large parameter set, the short-term pain is more than offset by welfare gains in the medium and long term.

5.3 Post-Policy Equilibrium and Transitional Dynamics Using the Small Parameter Set

Table 5 shows the results using the small parameter set. Comparing with the counterparts using the large parameter set in Table 4, health H_t grows much slower. The ratio of consumption to physical capital x_t falls much more. Productivity A_t is lower throughout time which suggests that when health parameters are all small, productivity increases very slowly. The tax rate τ_t is negligibly higher. The real interest rate r_t becomes even lower. The physical capital K_t decreases more. Consumption

Table 5: Equilibrium shift and transitional dynamics caused by the policy that real-locates labor from manufacturing to health sector to double health labor from 5% to 10% of the nation's workforce using the **small** parameter set

	Pre-policy equilibrium		Post-policy dynamics				
	$t = 0$	$t = j$	$t = 1$	$t = 10$	$t = 35$	$t = 50$	$t = \infty$
L_t	5%	10%	10%	10%	10%	10%	10%
H_t	1	1	1.001	1.007	1.016	1.018	1.021
x_t	16.94%	16.55%	16.55%	16.55%	16.55%	16.56%	16.56%
A_t	0.3	0.289	0.289	0.289	0.290	0.290	0.290
τ_t	0.3	33.07%	33.07%	33.07%	33.07%	33.07%	33.07%
r_t	6%	5.65%	5.65%	5.65%	5.65%	5.65%	5.65%
g_t	2%	1.58%	1.58%	1.58%	1.58%	1.58%	1.58%
K_t	6.667	6.667	6.773	7.807	11.597	14.707	∞
C_t	1.129	1.103	1.121	1.292	1.920	2.435	∞

Notes: j denotes jump, which means the post-policy initial value.

C_t drops more in the short term and increases less in the medium and long term.

More importantly, g_t immediately drops down to 1.58%, and remains at this level and fails to grow back throughout the time horizon. Comparing with its counterparts — 1.81% after half of a century and 1.85% at the post-policy equilibrium in Table 4 — economic growth is hurt by investing in health regardless of whether we use the large or the small parameter set. But it is hurt much more when health parameters are all small than when they are all large.

Regarding the welfare effect, relative to the pre-policy equilibrium, the percentage change of the discounted utility summed over time after policy remains negative after thirty-five years. This suggests that when all health parameters are small, the health investment makes households worse off.

Table 6: Sensitivity test of the weight (ξ) of health relative to consumption in utility by reporting the percentage change of discounted utility summed over time with policy relative to that without policy

Post-policy dynamics			
	$t = 1$	$t = 10$	$t = 35$
Large set $\beta = 0.95, \gamma = 0.09$			
$\xi = 14$	-0.39%	+54.09%	+73.10%
$\xi = 5$	-8.93%	+9.71%	+18.15%
Small set $\beta = 0.05, \gamma = 0.03$			
$\xi = 14$	-14.92%	+2.70%	+7.82%
$\xi = 5$	-17.71%	-11.86%	-10.32%
Middle set $\beta = 0.50, \gamma = 0.09$			
$\xi = 14$	-2.97%	+51.70%	+69.21%
$\xi = 5$	-11.51%	+7.32%	+14.27%
Middle set $\beta = 0.50, \gamma = 0.03$			
$\xi = 14$	-14.03%	+3.50%	+9.09%
$\xi = 5$	-16.82%	-11.06%	-9.05%

5.4 Further Sensitivity Tests: Varying ξ , β , and γ Individually

We now report how the policy's welfare effect is sensitive to the values of the three parameters ξ , β , and γ . Table 6 reports the percentage changes of discounted utility summed over time with the policy relative to that without the policy, when the value of health is large ($\xi = 14$) and when it is small ($\xi = 4$), for different combinations of the other two health parameters. Comparing the two rows within each panel, that is, fixing β and γ values, the size of ξ affects the welfare changes considerably both in size and sometimes even in sign.

Table 7 reports the sensitivity tests of β and γ by reporting the percentage change of discounted utility summed over time. To detect whether the size of the productivity-enhancing effect of health (β) matters in the welfare effect of the policy, we focus within the first and second panels respectively. When fixing $\gamma = 0.09$ and $\xi = 14$, the welfare gain after thirty-five years of policy is +73.10% when β is 0.95, +69.21% when β is 0.50, and +65.36% when β is 0.05. These values are not

Table 7: Sensitivity test of return to labor in health production (γ) and return to health in manufacturing productivity (β) by reporting the percentage change of discounted utility summed over time with policy relative to that without policy

Post-policy dynamics			
	$t = 1$	$t = 10$	$t = 35$
$\xi = 14, \gamma = 0.09$			
$\beta = 0.95$	-0.39%	54.09%	73.10%
$\beta = 0.50$	-2.97%	51.70%	69.21%
$\beta = 0.05$	-5.75%	49.27%	65.36%
$\xi = 5, \gamma = 0.03$			
$\beta = 0.95$	-15.95%	-10.27%	-7.78%
$\beta = 0.50$	-16.82%	-11.06%	-9.05%
$\beta = 0.05$	-17.71%	-11.86%	-10.32%

Post-policy dynamics			
	$t = 1$	$t = 10$	$t = 35$
$\xi = 5, \beta = 0.05$			
$\gamma = 0.09$	-14.29%	4.89%	10.41%
$\gamma = 0.03$	-17.71%	-11.86%	-10.32%
$\xi = 14, \beta = 0.50$			
$\gamma = 0.09$	-2.97%	50.70%	69.21%
$\gamma = 0.03$	-14.03%	3.50%	9.09%
$\xi = 5, \beta = 0.50$			
$\gamma = 0.09$	-11.51%	7.32%	14.27%
$\gamma = 0.03$	-16.82%	-11.06%	-9.05%
$\xi = 14, \beta = 0.95$			
$\gamma = 0.09$	-0.39%	54.09%	73.10%
$\gamma = 0.03$	-13.16%	4.29%	10.36%
$\xi = 5, \beta = 0.95$			
$\gamma = 0.09$	-8.93%	9.71%	18.15%
$\gamma = 0.03$	-15.95%	-10.27%	-7.78%

substantially different from one another. Similar patterns hold when fixing $\gamma = 0.03$ and $\xi = 5$ except that all signs reverse. This suggests that the contribution of health's productivity-enhancing effect to the policy's welfare effect is limited.

To detect whether the effectiveness of labor in producing health (γ) matters for the welfare effect of the policy, we compare within the third – seventh panels in Table 7 respectively. The third panel fixes $\beta = 0.05$ and $\xi = 5$, the welfare gain after thirty-five years of the policy is +10.41% when γ is large (0.09). But it turns to a welfare loss of -10.32% when γ is small (0.03). Similar reverse is found while fixing ξ small and β moderate (the fifth panel) and large (the seventh panel). Whereas welfare always gains regardless of the size of γ in the fourth and sixth panels. But the welfare gains are 8–16 times higher when γ is big (0.09) than the gains when γ is small (0.03).

Table 8: A simulation using parameter values suitable for Canada by reporting the percentage change of discounted utility summed over time with policy relative to that without policy

Post-policy dynamics			
	$t = 1$	$t = 10$	$t = 35$
	$\beta = 0.05, \gamma = 0.09$		
$\xi = 14$	-5.75%	+49.27%	+65.36%
$\xi = 5$	-14.29%	+4.89%	+10.41%

Summing up, the sensitivity tests in Tables 6 and 7 indicate three results. First, whenever β and γ are fixed, the size of health's relative weight (ξ) substantially affects the size or sign of the welfare effect of the policy. Second, no matter how large or small the effectiveness of labor in health production (γ) and the weight of health in utility (ξ) are, the size of the productivity-enhancing effect of health (β) does not substantially affect the welfare effect of the health investment policy. Third, no matter how large or small the productivity-enhancing effect of health (β) and the weight of health in utility (ξ) are, the magnitude of the effectiveness of labor in health production (γ) affects the welfare effect of the policy substantially, and it affects even the sign of the welfare effect when ξ is small. This suggests that, regardless of the relative values of health and consumption in utility, as long as the health investment has a sufficiently large return in generating health status, it generates welfare gains.

Table 8 shows a simulation using a set of parameter values that are likely to be the most suitable for Canada. In this simulation, γ is as large as 0.09 calculated from Canadian historical data, while β is as small as 0.05 since the aged population who need health services most are less likely to participate in the workforce. The welfare gains when ξ is large (14) are about 6–10 times higher than the gains when ξ is small (5) in the medium and long run. Meanwhile, the long-run economic growth rate decreases to 1.59% at the post-policy equilibrium.

6 Conclusions and Policy Implications

This paper examines the competing effects of an investment in health on welfare and growth. Given our applied policy focus, our model is designed to be relatively simple while still capturing important trade-offs in order to provide insights into a range of issues associated with the investment in health for policy makers.

We offer four major conclusions and policy implications. First, a policy that invests in health improves health status hence welfare in the long run, but harms the economic growth in both short and long term. The relative size of these competing effects depends on the specific health features of the country. When the productivity-enhancing effect of health and the effectiveness of labor in health production are both large, more than half of the immediate harm in economic growth recovers in the long run. That is, the economic slowdown is partially offset in the long run by the enhanced productivity. Such long-run welfare and growth effects we find are consistent with existing theoretical literature, in particular, with Van Zon and Muysken (2005) (p.58–60) who find similar results from a numerical analysis that is limited to comparing only full equilibrium outcomes. They are also consistent with the recent empirical work Aghion *et al.* (2010) who find that, for OECD countries, only the reduction in mortality below age forty generates productivity and growth enhancement. Therefore, our findings together with relevant literature challenge the policy rationales of the World Bank (1993) and the World Health Organization (2001) in the sense that good health, though it improves welfare, decreases the economic productivity of workers and the economic growth rate of countries.

Second, no matter whether we believe that households value health much higher than material consumption or not, households are always better off by the health investment policy as long as the effectiveness of health investment in producing health is large. Households can be worse off by such a policy only if both health investment is

not productive in generating health and households value health to a relatively small extent (5 times and less than consumption). In the real world at the disaggregate level, there can be many kinds of particular investments in health from the health sector such as medical professionals, hospital beds, and diagnostic tests, and from the non-health sectors such as public funding, improved housing, and education. Our results indicate that policy makers need to be aware of such a sensitivity of the finding to the effectiveness of health investments in producing health, so as to invest wisely to achieve the envisioned welfare gains.

Third, this paper’s emphasis on investment in the public health care sector applies to developed countries who have a publicly funded health sector like Canada and the U.K. For these countries where the productivity-enhancing effect of health is small but the effectiveness of labor in health production is still large, an investment in health is likely to lead to a welfare gain for households over the medium and long run. However, such an investment is likely to slow down economic growth in both the short and long run. Such a lowered economic growth shrinks the size of the pie of national income that would have been larger (than without the policy) for the government to redistribute to the poor and reduce the poverty.

Fourth, our model serves as a sensitivity test for the existing simulation studies. Compared to Van Zon and Muysken (2005), we indicate that the importance of relying on a health production function as traditionally documented in macroeconomic literature rather than population-based function as is their focus. Our model is also a sensitivity test for Hall and Jones (2007) in three ways. It does not rely on a strong interaction between health and consumption in households’ utility formulation as done in their basic model. It assumes Ricardian equivalence (the young is born to replace the old once they die, and the welfare of the young is of equal concern for the “family dynasty”) as opposed to their age-specific overlapping generations framework. Finally, it tests the sensitivity of the conclusions to varying assumptions about

the value of health compared to consumption. Overall, given our conclusion about the welfare gain of health investment as long as health labor is productive enough regardless of the relative weight of households' value on health, our model shows the robustness of the Van Zon and Muysken (2005) and Hall and Jones (2007)' findings.

Our model has limitations. We have assumed that all households are identical. Hence, we cannot discuss the relationship between inequality and economic growth in this model which would be interesting as emphasized by Aghion *et al.* (1999). The model can be modified to eliminate the use of linear approximations of all relationships, and this may have important effects. Turnovsky *et al.* (2010) find that in a one-sector endogenous growth model where governmental spending enters the production function, the reliability of a linear approximation depends on the nature of governmental spending and the size of the adjustment speed. They find that if the productive governmental spending is an additional state variable as is the case if that spending were focused on an accumulating stock such as infrastructure, then the error of linearly approximated transitional dynamics can be big. This is the case in our paper since health enters the manufacturing production as a state variable, so their analysis does raise a concern. Further, they find that the smaller the adjustment speed per annum, the bigger the linear approximation error. This is also a concern for our analysis since our annual adjustment speed is mere 4%. So there is certainly scope for further fruitful analysis. However, it must be remembered that virtually no macroeconomic model of health that is accessible to policy makers has considered transitional dynamics at all. So our analysis of transition paths is both a contribution in its own right (especially given that it challenges the recommendations of policy-oriented institutions) and an important stepping stone for future analyses.

Appendix

Transitional Dynamics Between Pre-Policy and Post-Policy Equilibria:

Given the assumption that the system is linearized, we analyze the transitional dynamics in the neighborhood of the post-policy full equilibrium. Solving the equations (1) and (14) for H_t and x_t , and setting further changes in L to zero, we have

$$d\dot{H} = v dH \text{ or } \dot{H} = -\delta(H - H^*) \quad (21)$$

which implies

$$H_t = H^* - (H^* - H_0)e^{vt}. \quad (22)$$

By substituting H^* from equation (18), H_0 , and v into equation (22), we obtain H_t for all time t , which is the transitional dynamics of health status.

Since both H and x in the dynamic system share the same speed of adjustment along the saddle path, we have the similar solution for x_t

$$x_t = x^* - (x^* - x_j)e^{vt}, \quad (23)$$

where x_j is the initial value of x *after* the policy since x is a jump variable. Notice that $x_j < x_0$ and $x_j > x_0$ are both possible. In other words, x_t can jump either down or up after the policy shock. Also, the new equilibrium value x^* can be lower or higher than x_0 . Such ambiguities require the numerical simulations in Section 5.

By substituting equation (19), x_j , and v into equation (23), we can get x_t if we know x_j . To find x_j , we rely on the fact that for a linearized system the equation of saddle path must be linear; that is,

$$x^* - x_t = b(H^* - H_t), \quad (24)$$

which also holds initially after the policy; that is,

$$x_j = x^* - b(H^* - H_0). \quad (25)$$

To find b , we rewrite equations (22) and (24):

$$H^* - H_t = (H^* - H_0)e^{vt}, \quad (26)$$

$$x^* - x_t = (x^* - x_0)e^{vt}. \quad (27)$$

We can rewrite our system (15) without the exogenous variable dL (which is appropriate for all time periods beyond $t = 0$, since L changes only at $t = 0$) as

$$\begin{bmatrix} \dot{H} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -\delta & 0 \\ \frac{(\alpha(1-\tau)-(1-z)+rq_2)\beta(1-\alpha)Ax}{H} & x \end{bmatrix} \begin{bmatrix} dH \\ dx \end{bmatrix}. \quad (28)$$

Substituting both equations (26) and (27) and their time derivatives into equations (28) gives

$$b = \frac{(\alpha(1-\tau)-(1-z)+rq_2)\beta(1-\alpha)A\bar{x}}{(-\delta-x)H}. \quad (29)$$

By substituting b back to equation (25), we get x_j which then can be substituted back to equation (23) to obtain x_t for all time t .

Given that we know $x_t = C_t/K_t$ for all time t , to find C_t we first need to find K_t . This is straightforward since, at each point in time, both A_t and x_t are given. We know that

$$\frac{\dot{K}}{K} = (1-z)A - x - \delta,$$

which implies

$$K_t = e^{((1-z)A-x-\delta)t} K_0, \quad (30)$$

where K_0 is the initial capital stock (the pre-policy equilibrium value). Equation (30)

can be used, along with the values we know for x_t and A_t (from equations (23) and (5)) to calculate K_t for all t beyond $t = 0$.

Then $C_t = x_t K_t$ is known by substituting x_t from equation (23) and K_t from equation (30). Corresponding to x_j , $C_j = x_j K_0$ is the initial value of jump variable C after the policy (the post-policy initial value).

After substituting C_t and H_t into a discrete-time version of equation (11)

$$U_t = \sum_{t=0}^f \left(\frac{1}{1+\rho} \right)^t \left[\ln C_t + \xi \ln H_t \right], \quad (31)$$

we obtain U_t for any chosen time horizon of f years into the future.

Alternative calibration of the effectiveness of labor in health production (γ) using Van Zon and Muysken (2005):

Another way to find the γ value follows from the analysis in Van Zon and Muysken (2005), which can be explained by referring to their Figure 2.1 (our Figure 1):

Figure 1: Flow chart of the healthy and sick population from Fig. 2.1 Van Zon and Muysken (2005)

This is a flow chart similar to the commonly-used Susceptibles-Recovery model in mathematical epidemiology (Brauer *et al.* (2008)). In explaining this model, we use the notation from Van Zon and Muysken (2005), so the variables differ from how we have defined these symbols in our model. The total population (P) is divided into two groups, the healthy (H) and the sick (S). A proportion (ι) of the healthy people give new births (B) to the healthy pool. Whereas among the sick, some (C) are cured back to full health, as a proportion (δ_0) of health labor which is further a proportion (v) of the healthy population. Some as a proportion (χ) of the sick (S) are currently cared by the other type of health labor (u) hence stay in sick. Others (D) die as a proportion (μ_x) of the sick. Besides, new cases (N) come from the healthy population as a proportion (μ_s) to the sick population. Thus, the dynamic system of the healthy and the sick can be summarized as below

$$\dot{H} = B + C - N = \iota H + \delta_0 v H - \mu_s H, \quad (32)$$

$$\dot{S} = N - C - D = \mu_s H - \delta_0 v H - \mu_x H. \quad (33)$$

These two equations are useful, because the goal is to find the steady-state population health h^* (that is, the steady-state proportion of the healthy population in the total population) at the equilibrium $\dot{h}/h = \dot{H}/H - \dot{P}/P = 0$. On the one hand, we can get $\dot{H}/H$ by dividing equation (32) by H . On the other hand, recall the total population P is the sum of the healthy and the sick, this implies $\dot{P} = \dot{H} + \dot{S}$ whose expression is obtained by substituting equations (32) and (33). By solving the resulting $\dot{h}/h = 0$ for h^* , we obtain equation (34) where the steady-state population health positively depends on the health labor who cure (v^*) sick people back to healthy.

We now turn to health labor who care (u) the sick people remaining in illness or dying. Such a demand for care services equals the supply of care services at the equilibrium; that is, $u^*h^*P = \chi(1 - h^*)P$. By solving this equation for u^* , we obtain equation (35). It states that the steady-state population health can also negatively depend on the health labor who care, which indicates an interesting trade-off between the curing (v^*) and caring (u^*) health labor. Finally, the curing and caring health labor are subject to the constraint of total health labor L in equation (36).

To summarize, Van Zon and Muysken (2005) have derived three equations of three endogenous variables h^* , v^* , and u^* from the system at the equilibrium

$$h^* = \frac{\delta_0}{\tau + \mu_x}v^* + \left(1 - \frac{\mu_s}{\tau + \mu_x}\right), \quad (34)$$

$$u^* = \chi \frac{1 - h^*}{h^*}, \quad (35)$$

$$L = v^* + u^*. \quad (36)$$

We can solve the three equations for

$$\frac{dH}{dL} = \frac{\frac{\delta_0}{\tau + \mu_x}}{1 - \frac{\chi\delta_0}{(\tau + \mu_x)(h^*)^2}},$$

which is equivalent to $dH/dL = \gamma H/L$ in our model. By substituting the assumed

parameter values in Van Zon (2005) Table 2.2, we obtain γ as 0.03.

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