Section B

Remember to use a different booklet to answer the questions in Section B.

1. Let $d : X \times X \to \mathbb{R}$ be a metric on a set X. Show that

$$|d(x, x') - d(x, x'')| \leq d(x', x'')$$

for any x, x', and x'' in X.

2. Consider the sequence $\{x_k\}$ defined by

$$x_1 = 0 \text{ and } x_{k+1} = \sqrt{x_k + 6} \text{ for all } k = 1, 2, 3, ...$$

   (a) Prove that the sequence is bounded.

   (b) Prove that the sequence is increasing.

   (c) What is the limit of the sequence?

3. Determine whether the following function is quasiconcave or quasiconvex or neither

   (a) $f(x, y) = x^2 + xy \quad (x, y > 0)$

   (b) $f(x, y) = xy^2 \quad (x, y > 0)$

4. Suppose that a decision maker wants to find out (x_1, x_2) that maximizes $f(x_1, x_2) = ax_1 + \ln x_2$ subject to $x_1 + qx_2 \leq p$, $x_1 \geq 0$, and $x_2 \geq 0$, where $a > 0$, $p > 0$ and $q > 0$. Derive the solution.

- End of Exam -