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International Consequences of Population Ageing: Towards a Three-Region Overlapping Generations Model

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Abstract

A three-region three-good fifteen overlapping generations model is built to analyze the international consequences of population ageing in the world. Each region produces only one good, but foreign goods are perceived as imperfectly substitute to the domestic good. Households work the first 12 periods of their life and retire for the last three. They behave in a life-cycle way with respect to consumption and savings. After optimizing a utility function composed of an aggregate good and bequests, they allocate their current consumption between domestic and the two foreign goods. They also invest in the domestic and foreign market, allocating regionally their savings between government bonds and capital. Regions differ by their demographic projections. As shown in Buiter (JPE 1981), a stationary state exists with constant current account imbalances/GDP ratios when regions differ only by the rate of time preference (demographic structure). In each region a pay-as-you go pension scheme is financed through general taxation revenue. Besides that, government has the usual budget constraint. The paper investigates different scenario regarding government policies associated with pressures arising from population ageing.

1. Motivation

A significant shift in the makeup of the world`s population is under way that could make one of the two (the other probably being environmental issues) transcendent political and economic issues of the XXIst century. The change is the ageing populations in all the developed and many of the developing nations of the world. It will see those over 65 years of age, who historically made up no more than 2 to 3 percent of the total population and now amount to about 14 percent of current time developed world, becoming 25-30 percent of the population within another 30 years.

The world population is ageing, but at different paces, and according to the UN demographic projections, things should go on like this well into the XXIst century. The social and economic consequences of ageing from a global perspective have already deserves some attention. The Group of Ten, in its 1998 report, has raised concerns on the implications for current account balances of many countries. Higgins (1998) supports that view. In his paper he estimates that the demographic effect on the current account balance exceeds 6 percent of GDP over the last three decades for a number of countries. He expects this number to be substantially larger over the coming decades. Summers (2000) stresses the need to build an adequate international financial system to permit the exceptional opportunity of mutual gains from intertemporal trade, as most of the world`s population growth over the next 25 years will take place in the developing countries. He hopes that the international architecture of the financial system will permit the transfer of capital, expertise and know-how, from the developed world to the developing world. Despite this interest, modelisation efforts on population ageing have so far mainly concentrated on single country issues. Very few analyses have explicitly taken the worldwide aspect of the problem into account. One exception is Turner *et al* (1998) who develop a multi-country overlapping generations model under the Blanchard's constant-planning-horizon version. The life-cycle behaviour, which is central to any investigation of population ageing is imposed in an ad hoc manner. Ingenue Team (2000) is to our knowledge the first attempt to develop an international overlapping generations model with fully life-cycle features that investigate the different prospects between developed and developing countries of asset accumulation and of international capital flows over the next few decades. Their work is still (like ours) at a preliminary stage.

Our paper concentrates on the developed world. The model we propose is a three-region computable general equilibrium with an overlapping generations structure in the Auerbach-Kotlikoff tradition. The three regions are Europe, the United States and Japan. Japan is the most advanced in its population ageing process and Europe is expected to age more rapidly than the United States. As these regions trade substantially among themselves, the difference in pace and intensity of the population ageing process will certainly influence their current account patterns. The economic size of these three regions will also continue to determine worldwide interest rates. Moreover, it is likely that the United States increase the funding of their social security programs. If capital increases as a result of pension reforms in the United States, interest rates may decline and wage rates may increase. This may alleviate the population ageing pressure on pay-as-you go pension programs in Europe.

2. The Model

The overlapping generations structure of the model is similar to Fougère and Mérette (1999a, 1999b). The model is based on the life-cycle theory of savings behaviour. In the models, there are 15 generations living side by side at each point in time. Each new generation has 15 periods to live, with each period corresponding to 4 years of life. The 15 generations included in the model are 16 to 75 years of age. Individuals are assumed to work until age 63, so 12 of the 15 generations are members of the active population. Population growth rates are exogenous. In comparison to the seminal work Auerbach and Kotlikoff (1987), labour supply is exogenous but bequest motives are included.

Technologies and Firm Behaviour

The final goods sector production depends on physical capital and effective labour. All firms are identical. The economy's production technology is represented by a Cobb-Douglas function:

(1) $$Y_t = AK_t^{\varepsilon} L_{e,t}^{1-\varepsilon} \quad ,$$

where Y is real final output, A a scaling variable, K the stock of physical capital, L_e effective labour and ε the capital income share.

The productivity of a worker (*ep*) is assumed to be age-related following this quadratic function of the age *g*:

(2) $$ep^g = \gamma + \lambda g - \psi g^2, \ \gamma, \lambda, \text{ and } \psi > 0.$$

Furthermore, it is assumed that technical change is exogenous and "labour embodied"; every new generation has a larger stock of technical knowledge than the previous cohort and is more productive by a constant factor η. Thus, the supply of effective labour is growing over time with a constant rate in excess of the growth rate of the working-age population. Consequently, the effective productivity of each age-group at period t is $ep^g\,\eta_t^g$, and effective labour supply at period t equals:

$$(3) \qquad L_{e,t} = \sum_{g=1}^{12} ep^g \eta_t^g \, pop_t^g \ ,$$

where $pop^g{}_t$ is the number of people of age g at period t.

Factor demands stem from profit maximisation by firms. Firms rent physical capital at the market rental rate and hire labour at the market wage rate per unit of effective labour, up to the point at which their marginal products equal their marginal costs:

$$(4) \qquad \frac{rent_t - \delta}{p_t} = \varepsilon A K_t^{\varepsilon-1} L_{e,t}^{1-\varepsilon} \quad ,$$

$$(5) \qquad \frac{w_t}{p_t} = (1-\varepsilon) A K_t^{\varepsilon} L_{e,t}^{-\varepsilon} ,$$

where r is the rental rate (net of depreciation) of capital, w the wage rate, p the ouput price and δ the rate of capital depreciation. The firm's wage bill is thus the product of the gross effective wage rate times the stock of effective labour supplied by all living individuals.

Cohort Behaviour

There is a representative individual for each generation. Each generation maximises its utility function, U, of consumption and bequest given its lifetime income.

The representative generation's preferences are represented by the following constant intertemporal elasticity of substitution utility function:

(6) $$U = \frac{1}{1-\theta}\sum_{g=1}^{15}\left(\frac{1}{1+\rho}\right)^{g}\left(c_g^{1-\theta} + \beta_g^{\theta} Beq_g^{\;1-\theta}\right), \quad 0 < \theta < 1; \; \beta_{g\neq 15} = 0, \; \beta_{g15} > 0,$$

where c is the composite consumption good, ρ the pure rate of time preference, θ the inverse of the intertemporal elasticity of substitution, β is a constant parameter and *Beq* is bequest. Bequest motives are specified as in Blinder (1974). Bequests are distributed at the end of each generation's lifetime and are received equally by all working generations. A generation's lifetime profile of wage income is calibrated in a similar way for all models. Each generation's interest income is determined by its stock of physical wealth.

The One-period Budget Constraint

The representative generation budget constraint at each period is:

(7) $$\begin{aligned} pcon_t a_{t+1}^{g+1} &\leq w_t ep_t^g \eta_t^g (1-\tau_{w,t}) + rb_{t-1} bd_t^g (1-\tau_k) + rk_t pinv_{t-1} kd_t^g (1-\tau_k) \\ &- pcon_t con_t^g + pen_t^g \left(1-\tau_{w,t}\right) + pcon_t \left(Inh_t - Beq_t\right) \end{aligned},$$

where a is physical wealth asset; *bd* and *kd* are bond and physical capital holdings respectively, τ_w, τ_k and τ_c are tax rates on labour and pension incomes, interest income and consumption, respectively; *pcon* and *pinv* are the price index of aggregate consumption and physical capital respectively; *rb* and *rk* are rate of returns to bonds and physical capital respectively; *pen* are pensions and *Inh* inheritances. Inheritances are received equally by the working generations:

(8) $$Inh_t^j pop_t^j = \frac{1}{12} Beq_t^m pop_t^m \quad , \; j = 1,2,\ldots,12 \; and \; m = 15.$$

Finally, the amount of pensions received depend on the replacement rate α of previous wage earnings:

(9) $$pen_t^m = \alpha \cdot \frac{1}{12}\sum_{g=1}^{12} w_{t-m+g}\, ep^g \eta_{t-m+g}^g \text{ , } m=13,14,15.$$

Physical capital and bonds markets are assumed efficient. Competitive markets ensure that their exante rate of returns are equal (*rb=rk*). In the model we however need an allocation rule of the household portfolio between physical capital and government bonds assets. We assume that the household's share of total physical capital in his economy equals his share of wealth with respect to the total wealth in the economy. That is,

(10) $$\frac{kd_{t+1}^g}{K_{t+1}} = \frac{pcon_t a_{t+1}^{g+1}}{\sum_g pop_t^g\, pcon_t a_{t+1}^{g+1}}$$

Consequently, the househol's holding of bonds is equal to:

(11) $$bd_{t+1}^{g+1} = pcon_t a_{t+1}^{g+1} - pinv_t kd_{t+1}^{g+1}$$

The representative cohort considers consumption from different countries as imperfect substitutes (Armington assumption). Household's preferences with respect to geographic origin are represented by a constant elasticity of substitution function (*CES*). The optimal composition of its consumption basket with respect to the three region origins is given by the solution of the following optimisation problem:

(12) $$Min_{c_{i,j}} \quad \sum_i (1+\tau_c) p_i c_{i,j} = pcon_j con_j$$

subject to $con_j = \left[\sum_i \alpha^c_{i,j} c_{i,j}^{\frac{\sigma^c - 1}{\sigma^c}} \right]^{\frac{\sigma^c}{\sigma^c - 1}}$

Physical capital is a composite goods of the three goods available. There thus exists an optimal investment allocation across regions that can be formulated as:

(13) $Min_{c_{i,j}} \quad \sum_i p_i invi_{i,j} = pinv_j inv_j$

subject to $inv_j = \left[\sum_i \beta_{i,j} invi_{i,j}^{\frac{\sigma^{inv} - 1}{\sigma^{inv}}} \right]^{\frac{\sigma^{inv}}{\sigma^{inv} - 1}}$

Government Behaviour

The pension system is assumed to be pure "pay-as-you-go" (PAYG) and is fully integrated into government accounts. The government has the responsibility to maintain the solvency of the pension fund by obtaining sufficient contributions from each generation. The government needs to finance public expenditures and pensions, using domestic bond issues and taxation. The one-period budget constraint of the government is given by:

(10) $B_{t+1} - B_t = rb_t B_t + p_t G_t + PEN_t - T_t$,

where B are government bonds, G government expenditures, PEN is total pensions payments ($PEN = \sum_g pop^g pen^g$) and T government revenues out of taxation.

The public-sector debt-to-GDP ratio is assumed to be constant and the PAYG pension plan is financed through a wage-income tax. Government expenditures are restricted to pensions, spending on public goods and interest payments on the public debt. Public good expenditures affect neither private consumption nor production in the model. The government collects three types of taxes from each generation: on wage income, capital income and consumption. Tax rates on capital income and consumption are kept exogenous. The government's debt instruments are one-period bonds that pay the prevailing market interest rate in the current period and the principal in the next period. Summing over all generations, total government revenue is:

$$\begin{aligned} T_t = \tau_{w,t} \sum_g \left(w_t ep^g n_t^g + pen_t^g \right) pop_t^g + \tau_k rb_t \sum_g bd_{k,t}^g \cdot pop_t^g + \\ \tau_k rk_t \sum_g pinv_{t-1} kd_{k,t}^g \cdot pop_t^g + \tau_c \sum_g pcon_t con_t^g \cdot pop_t^g \end{aligned} \quad (11)$$

Capital Market

In an open economy framework, physical capital plus government debt equals total private wealth plus the stock of net foreign debt every periods:

$$K_t + B_t = \sum_g a_t^g \cdot pop_t^g + NFD_t \quad , \quad (12)$$

where *NFD* is the stock of net foreign debt. In addition, final goods output equals household and government consumption (*C*+*G*), plus net investment I_t^N and the current account balance *NX*:

$$Y_t = C_t + G_t + INV_t + NX_t \quad , \quad (13)$$

where $C_{j,t} = \sum_i \sum_g c_{i,j,t}^g pop_{i,t}^g$, $INV_t = K_{t+1} - (1-\delta)K_t$ and

$NX_t = -(NFD_{t+1} - NFD_t - r_t NFD_t)$.

The bond market is assumed globally integrated. Hence the equilibrium condition requires that total world demand equals total world emissions of government bonds, that is:

(20) $$\sum_j B_{j,t} = \sum_j \sum_g pop_{j,t}^g bd_{j,t}^g :$$

Capital market effeciency ensures that:

(22) $$rb_t = rk_{j,t+1} ,$$

where the return to physical capital is related to the rental rate in the following way:

(23) $$rk_{j,t} = \frac{pinv_t \left(rent_{j,t} + (1 - dep)\right)}{pinv_{t-1}}$$

As Buiter (1981) shows, current account imbalances are possible along a balanced growth path in a one-good overlapping generations model. He demonstrates that to generate current account imbalances in a two-period two-country version, it is only necessary to assume that the countries differ in their pure rate of time preference. In a many-period model, such as the one used in this paper, a domestic ageing process that diverges from the rest of the world is equivalent for saving behaviour to differences in pure rates of time preference. A faster ageing country behaves as if its "representative agent" was relatively more impatient. Consequently, such a country will experience a larger decline in private savings as does, in Buiter (1981), the country with the greater rate of time preference.

Appendix: The computable program equations

Households

1.budget constraint:

$$pcon_t a_{t+1}^{g+1} \leq w_t ep_t^g \eta_t^g (1-\tau_{w,t}) + rb_{t-1} bd_t^g (1-\tau_k) + rk_t pinv_{t-1} kd_t^g (1-\tau_k)$$
$$- pcon_t con_t^g + pen_t^g \left(1-\tau_{w,t}\right) + pcon_t \left(Inh_t - Beq_t\right)$$

2. Household`s holdings of physical capital:

$$\frac{kd_{t+1}^g}{K_{t+1}} = \frac{pcon_t a_{t+1}^{g+1}}{\sum_g pop_t^g pcon_t a_{t+1}^{g+1}}$$

3. Household`s holdings of bonds:

$$bd_{t+1}^{g+1} = pcon_t a_{t+1}^{g+1} - pinv_t kd_{t+1}^{g+1}$$

4. Intertemporal first-order condition:

$$\frac{con_{t+1}^{g+1}}{con_t^g} = \left[\frac{rb_t pcon_t}{(1+\rho) pcon_{t+1}}\right]^{\sigma}$$

5. Consumption from different countries

$$c_{i,j,t}^g = \alpha_{i,j}^c \left[\frac{pcon_{j,t}}{p_{i,t}}\right]^{\sigma^c} con_{j,t}^g$$

$$pcon_{j,t}^{1-\sigma^c} = \left[\sum_i \alpha_{i,j}^c p_{i,t} \left(1+\tau_c\right)\right]^{1-\sigma^c}$$

6. Aggregate consumption and investment

$$Econ_{i,j} = \sum_g pop_{j,t}^g c_{i,j,t}^g$$

$$Einv_{i,j} = \sum_g pop_{j,t}^g invi_{i,j,t}^g$$

7. Bequests

$$Beq_{j,t}^{g} = \beta_{j}^{g} con_{j,t}^{g}$$

8. Pensions

$$pen_{t}^{m} = \alpha \cdot \frac{1}{12} \sum_{g=1}^{12} w_{t-m+g} ep^{g} \eta_{t-m+g}^{g} \text{, } m=13,14,15.$$

9. Inheritances

$$Inh_{t}^{j} pop_{t}^{j} = \frac{1}{12} Beq_{t}^{m} pop_{t}^{m} \quad , \quad j = 1,2,\dots,12 \;\; and \;\; m = 15.$$

10. Investments demand for different countries

$$invi_{i,j,t}^{g} = \alpha_{i,j}^{inv} \left[\frac{pinv_{j,t}}{p_{i,t}} \right]^{\sigma^{inv}} inv_{j,t}^{g}$$

$$pinv_{j,t}^{\;1-\sigma^{c}} = \left[\sum_{i} \alpha_{i,j}^{inv} p_{i,t} \right]^{1-\sigma^{inv}}$$

<u>Producers</u>

10. Technology

$$Y_{t} = AK_{t}^{\varepsilon} L_{e,t}^{1-\varepsilon}$$

11. Demand for labour

$$\frac{w_{t}}{p_{t}} = (1-\varepsilon) AK_{t}^{\varepsilon} L_{e,t}^{-\varepsilon}$$

12. Demand for capital

$$\frac{rent_{t} - \delta}{p_{t}} = \varepsilon AK_{t}^{\varepsilon-1} L_{e,t}^{1-\varepsilon}$$

13. Definition of rate of return on capital

$$rk_{j,t} = \frac{pinv_{j,t}\left(rent_{j,t} + (1-\delta)\right)}{pinv_{j,t-1}}$$

Government

1.Budget constraint

$$B_{t+1} - B_t = rb_t B_t + p_t G_t + PEN_t - T_t$$

$$PEN_t = \sum_g pop_t^g pen_t^g$$

$$T_t = \tau_{w,t} \sum_g \left(w_t ep^g n_t^g + pen_t^g\right) pop_t^g + \tau_k rb_t \sum_g bd_{k,t}^g \cdot pop_t^g +$$
$$\tau_k rk_t \sum_g pinv_{t-1} kd_{k,t}^g \cdot pop_t^g + \tau_c \sum_g pcon_t con_t^g \cdot pop_t^g$$

Equilibrium Conditions

1. Investment

$$rb_t = rk_{t+1}$$

2. Capital

$$K_{j,t+1} = INV_{j,t} + (1-\delta)K_{j,t}$$

3. Bonds

$$\sum_j B_{j,t} = \sum_j \sum_g pop_{j,t}^g bd_{j,t}^g$$

3. Goods

$$Y_{j,t} = \sum_i \left(Econ_{j,i,t} + Einv_{j,i,t}\right) + G_{j,t}$$